The Calculus of Temporal Influence

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Backstory: Digitalisation in (Secondary) Education

setting: natural science classes in secondary education

goal: learning tool that allows reasoning about experiments

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- formalizes the way different variables (temperature, yeast activity, . . .) interact

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$\rightsquigarrow$ precludes formalization of many natural phenomena

Solution: Introduce TIME

Influence Experiments

("signature" =) finite set of variables $\mathcal{V} = \{a, b, \ldots\}$, e.g. $\mathsf{glucose}, \mathsf{light}, \mathsf{volt}, \ldots$

study: functions of type $\mathsf{t} \to a$, where a is a variable, t is time

By convention:

- domain of time (t) is $\mathbb{R}_{\geq 0}$
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s.t.

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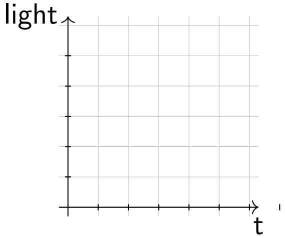

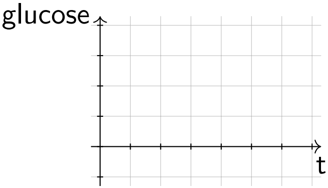

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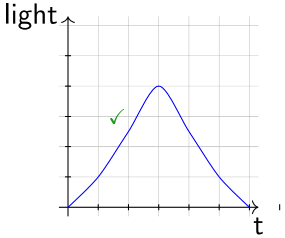

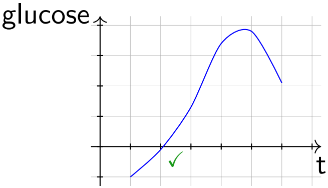

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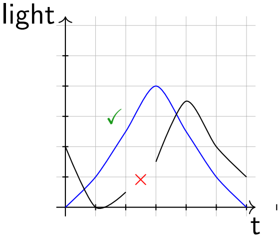

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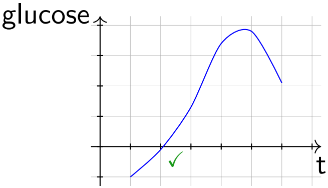

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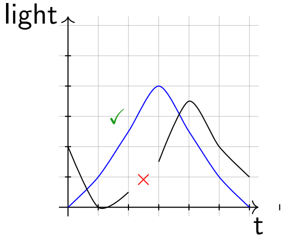

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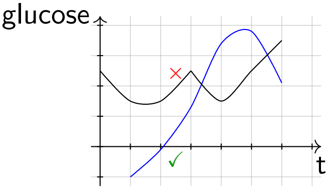
- $\mathcal{F}_a$ is continuously derivable on its domain



Time-Value Statements and Influence Schemes

Def.: time-value statement (TVS) S: $\mathsf{t} \xhookrightarrow{[t_1,t_2],I_1,I_2} a$ where

- a is a variable
- $[t_1, t_2]$, I_1, I_2 are intervals (bounds in $\mathbb{Q}$, mostly closed), e.g. $[0, 1]$, $[-10, 10]$, $[42, \infty]$, and $0 \leq t_1 \leq t_2$

intuitive meaning by example:

1. *"Between hours 2 and 4, light intensity is between 20% and 40%."*

   $\rightsquigarrow$ $\mathsf{t} \xhookrightarrow{[2,4],[20,40],[20,40]} \mathsf{light}$

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3. *"Between hours 5 and 6, altitude starts above flight level 100 and increases to a value above flight level 200"*
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Def.: influence scheme $\mathcal{C}$ = set of time-value statements, time-derivative statements, and value-derivative statements

Semantics of Time-Value Statements

Def. interpretation of TVS $S = \text{t} \xrightarrow{[x,y],[l,u],[l',u']} a$:

$\mathcal{F} \models S$ iff

- $\mathcal{F}_a(t) \leq u + (u' - u) \cdot \frac{t-x}{y-x}$ for all $t \in [x, y]$, and
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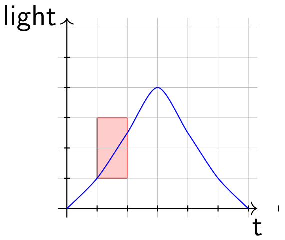
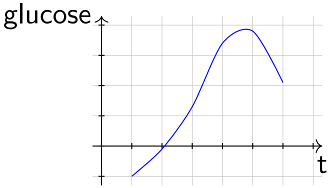
special case of $l = l'$, $u = u'$:

- $l \leq \mathcal{F}_a(t) \leq u$ for all $t \in [x, y]$.

Examples:

- $\mathcal{F} \models \text{t} \xrightarrow{[1,2],[1,3],[1,3]} \text{light}$





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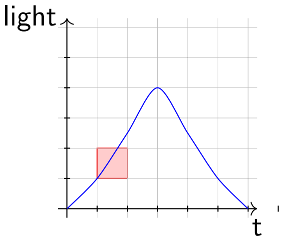

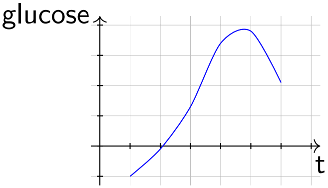

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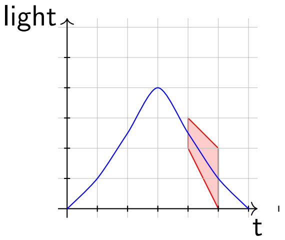

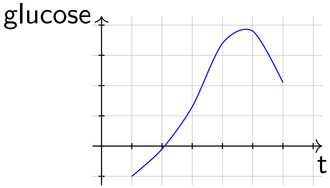

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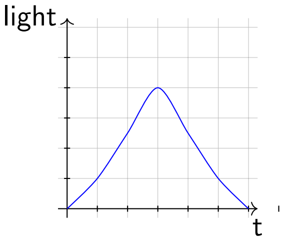

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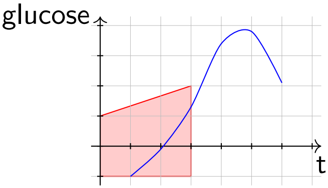

Time-Derivate Statements

$\mathcal{F}_a$ continuously derivable on its domain $\rightsquigarrow$ derivative $\dot{\mathcal{F}}_a$ defined and continuous

Def.: time-derivative statement (TDS) S: $\mathsf{t} \xhookrightarrow{[t_1,t_2],I} \dot{a}$ where

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Example:

1. *"Between hours 2 and 4, light intensity increases by at most 10% per hour."*
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2. "*Altitude never increases by more than 10 flight levels per hour*"
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Def. interpretation of TDS $S = \mathrm{t} \xhookrightarrow{[x,y],[l,u]} \dot{a}$:

$$\mathcal{F} \models S \text{ iff } l \leq \dot{\mathcal{F}_a}(t) \leq u \text{ for all } t \in [x, y].$$

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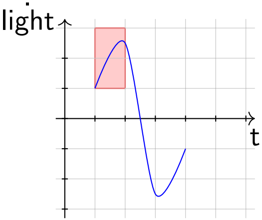

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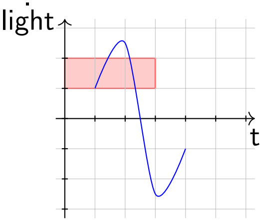

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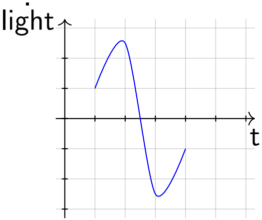

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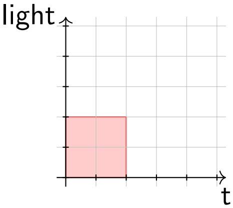

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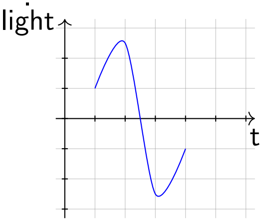

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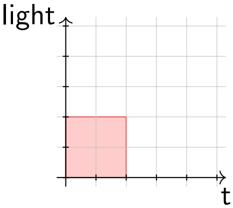

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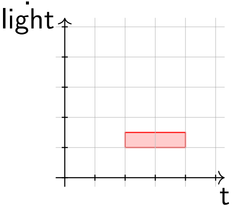

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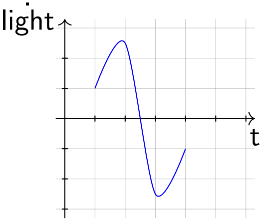

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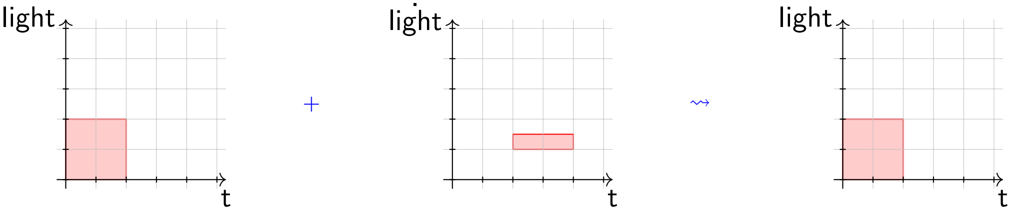

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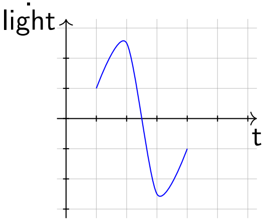
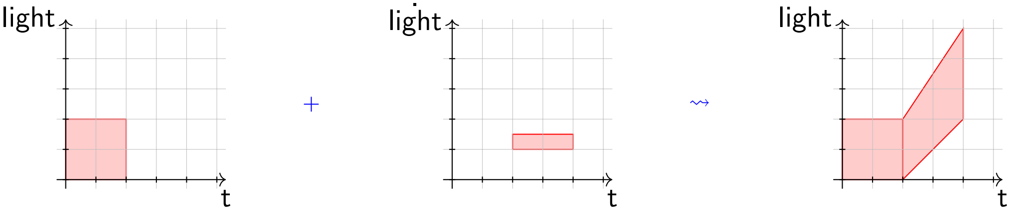


Examples:

- $\mathcal{F} \models \mathrm{t} \xrightarrow{[1,2],[1,3]} \dot{\text{light}}$
- $\mathcal{F} \not\models \mathrm{t} \xrightarrow{[0,3],[0,3]} \dot{\text{light}}$

(informal) consequence relation between TDS and TVS:



what about the other direction?

Variable-Derivative Statements

Dependencies between variables and derivatives are domain-specific:

Ex.: high light intensity means high glucose production in cell respiration

$\rightsquigarrow$ influences derivative of glucose level

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Example:

1. If light intensity is between 40% and 60%, then in the next hour, glucose levels rise by 5-25 units per hour $\rightsquigarrow$ $\text{light} \xhookrightarrow{[40,60],[0,1],[5,25]} \dot{\text{glucose}}$

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Def.: influence scheme $\mathcal{C}$ = set of time-value statements, time-derivative statements, and value-derivative statements

Semantics of Variable-Derivate Statements

Def. interpretation of VDS

$S = a \xhookrightarrow{[l,u],t_1,t_2],[l',u']} \dot{b}$: $\mathcal{F} \models S$ iff

$l' \leq \dot{\mathcal{F}}_b(t) \leq u'$ for all t s.t. there is t' with $\mathcal{F}_a(t') \in [l, u]$ and $t \in [t + t_1, t + t_2]$

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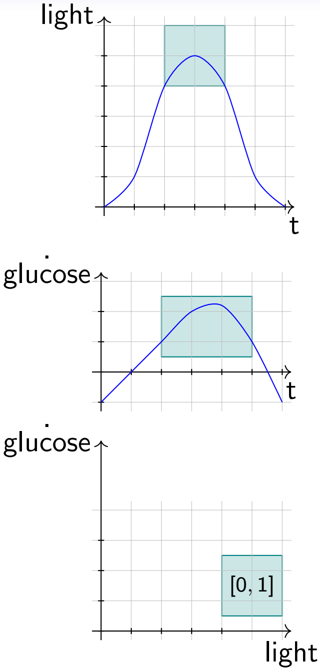

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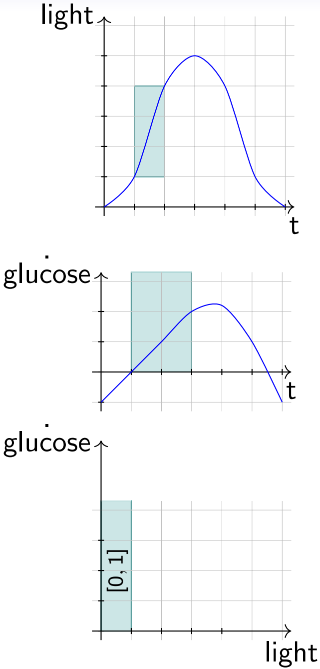

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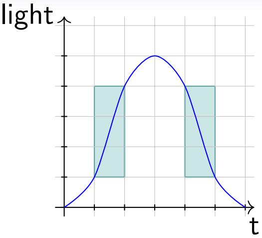
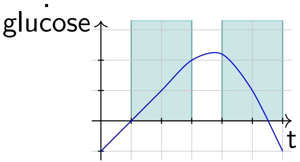
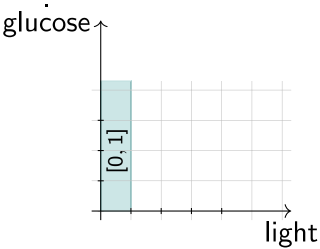
- only VDS effectively advance time
- VDR conceptually hard to grasp, given target audience
- will come back to this







The Calculus of Temporal Influence

Def.: (as usual) $\mathcal{C} \models S$ if for all $\mathcal{F}$: if $\mathcal{F} \models T$ for all $T \in \mathcal{C}$ then $\mathcal{F} \models S$

goal: proof-theoretic characterisation, $\mathcal{C} \vdash S$ iff S can be derived via ...

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(F) $$\frac{}{S} \quad \text{if } S \in \mathcal{C}$$

(VD) $$\frac{\mathrm{t} \xhookrightarrow{I_1,I_2,I_3} a}{\mathrm{t} \xhookrightarrow{I_1,[-\infty,\infty]} \dot{a}}$$

(DV) $$\frac{\mathrm{t} \xhookrightarrow{I_1,I_2} \dot{a}}{\mathrm{t} \xhookrightarrow{I_1,[-\infty,\infty]} a}$$

(Der) $$\frac{\mathrm{t} \xhookrightarrow{[t_1,t_2],I_1} a \qquad a \xhookrightarrow{I_1,[t_1',t_2'],I_2} \dot{b}}{\mathrm{t} \xhookrightarrow{[t_1+t_1',t_2+t_2'],I_2} \dot{b}}$$

(CDL) $$\frac{\mathrm{t} \xhookrightarrow{[t_2,t_3],I_1,I_2} a \qquad \mathrm{t} \xhookrightarrow{[t_1,t_2],I_3} \dot{a}}{\text{lderivative}([t_1,t_2],I_1,I_3)}$$

(G_{TDS}) $$\frac{\mathrm{t} \xhookrightarrow{[t_1,t_2],I_2} \dot{a} \qquad \mathrm{t} \xhookrightarrow{[t_1',t_2],I_2'} \dot{a}}{\mathrm{t} \xhookrightarrow{[t_2,t_1'],[-\infty,\infty]} \dot{a}} \quad \text{if } t_2 < t_1'$$

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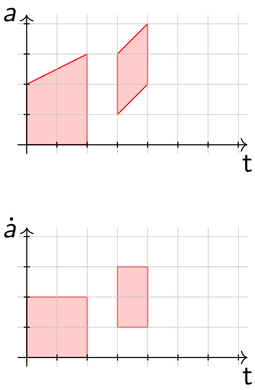
(S_{TDS}) $$\frac{\mathrm{t} \xhookrightarrow{I_1,I_2} \dot{a} \qquad \mathrm{t} \xhookrightarrow{I_1',I_2'} \dot{a}}{\mathrm{t} \xhookrightarrow{I_1 \cap I_1',I_2 \cap I_2'} \dot{a}}$$

(CDR) $$\frac{\mathrm{t} \xhookrightarrow{[t_1,t_2],I_1,I_2} a \qquad \mathrm{t} \xhookrightarrow{[t_2,t_3],I_3} \dot{a}}{\text{rderivative}([t_2,t_3],I_2,I_3)}$$

. . . and four analogous (but more complex) rules: (G_{TVS}), (W_{TVS}), (W_{TVS}), (S_{TVS})

The Calculus of Temporal Influence II

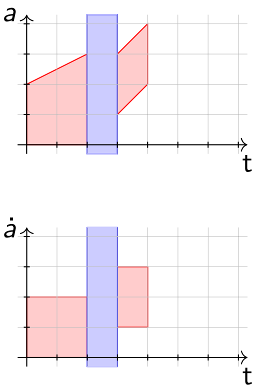
$$(\mathrm{G_{TDS}})\ \frac{\mathsf{t} \xhookrightarrow{[t_1,t_2],I_2} \dot{a} \qquad \mathsf{t} \xhookrightarrow{[t_1',t_2],I_2'} \dot{a}}{\mathsf{t} \xhookrightarrow{[t_2,t_1'],[-\infty,\infty]} \dot{a}} \text{ if } t_2 < t_1'$$



Rule $(\mathrm{G_{TDS}})$ not shown.

The Calculus of Temporal Influence II

$$(\mathsf{G}_{\mathsf{TDS}})\ \frac{\mathsf{t} \xhookrightarrow{[t_1,t_2],I_2} \dot{a} \qquad \mathsf{t} \xhookrightarrow{[t_1',t_2],I_2'} \dot{a}}{\mathsf{t} \xhookrightarrow{[t_2,t_1'],[-\infty,\infty]} \dot{a}}\ \text{if } t_2 < t_1'$$



Rule $(\mathsf{G}_{\mathsf{TDS}})$ not shown.

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$$(\mathrm{G_{TDS}})\ \frac{\mathsf{t} \xhookrightarrow{[t_1,t_2],I_2} \dot{a} \qquad \mathsf{t} \xhookrightarrow{[t_1',t_2],I_2'} \dot{a}}{\mathsf{t} \xhookrightarrow{[t_2,t_1'],[-\infty,\infty]} \dot{a}} \text{ if } t_2 < t_1'$$

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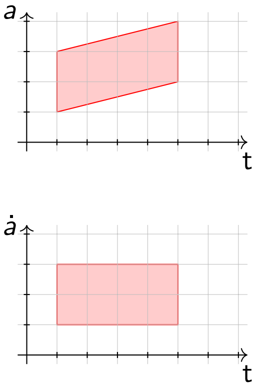

Rules $(\mathrm{G_{TDS}})$, $(\mathrm{W_{TVS}})$ not shown.

The Calculus of Temporal Influence II

$$(G_{TDS})\ \frac{t \xhookrightarrow{[t_1,t_2],I_2} \dot{a} \qquad t \xhookrightarrow{[t_1',t_2],I_2'} \dot{a}}{t \xhookrightarrow{[t_2,t_1'],[-\infty,\infty]} \dot{a}} \text{ if } t_2 < t_1'$$

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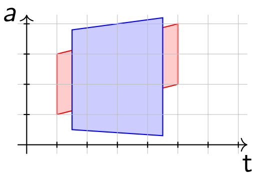

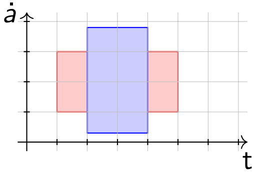

Rules (G_{TDS}), (W_{TVS}) not shown.

The Calculus of Temporal Influence II

$$(\mathrm{G_{TDS}})\ \frac{\mathrm{t} \xhookrightarrow{[t_1,t_2],I_2} \dot{a} \qquad \mathrm{t} \xhookrightarrow{[t'_1,t_2],I'_2} \dot{a}}{\mathrm{t} \xhookrightarrow{[t_2,t'_1],[-\infty,\infty]} \dot{a}} \text{ if } t_2 < t'_1$$

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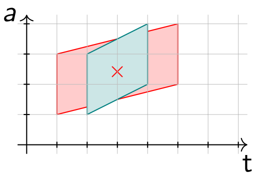

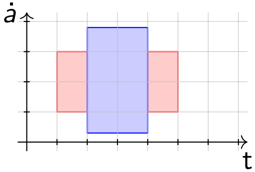

Rules $(\mathrm{G_{TDS}})$, $(\mathrm{W_{TVS}})$ not shown.

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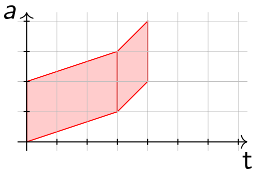

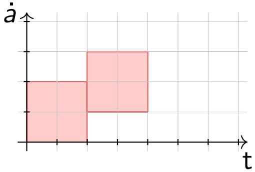

Rules $(\mathrm{G_{TDS}})$, $(\mathrm{W_{TVS}})$, $(\mathrm{J_{TDS}})$ not shown.

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$$(\mathrm{G_{TDS}})\quad \frac{\mathrm{t} \xhookrightarrow{[t_1,t_2],I_2} \dot{a} \qquad \mathrm{t} \xhookrightarrow{[t_1',t_2],I_2'} \dot{a}}{\mathrm{t} \xhookrightarrow{[t_2,t_1'],[-\infty,\infty]} \dot{a}} \quad \text{if } t_2 < t_1'$$

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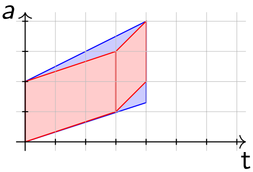

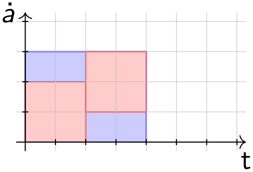

Rules $(\mathrm{G_{TDS}})$, $(\mathrm{W_{TVS}})$, $(\mathrm{J_{TDS}})$ not shown.

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$$(\mathrm{G_{TDS}})\ \frac{\mathsf{t} \xhookrightarrow{[t_1,t_2],I_2} \dot{a} \qquad \mathsf{t} \xhookrightarrow{[t_1',t_2],I_2'} \dot{a}}{\mathsf{t} \xhookrightarrow{[t_2,t_1'],[-\infty,\infty]} \dot{a}}\ \text{if } t_2 < t_1'$$

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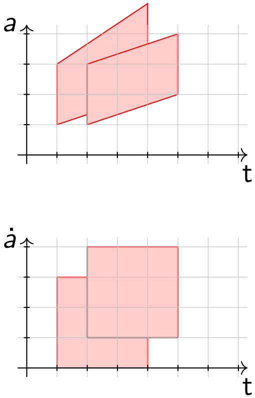

Rules $(\mathrm{G_{TDS}})$, $(\mathrm{W_{TVS}})$, $(\mathrm{J_{TDS}})$, $(\mathrm{S_{TDS}})$ not shown.

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$$(G_{TDS})\quad \frac{t \xhookrightarrow{[t_1,t_2],I_2} \dot{a} \qquad t \xhookrightarrow{[t_1',t_2],I_2'} \dot{a}}{t \xhookrightarrow{[t_2,t_1'],[-\infty,\infty]} \dot{a}} \text{ if } t_2 < t_1'$$

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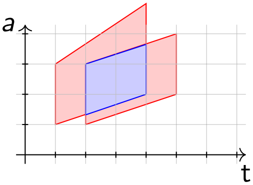
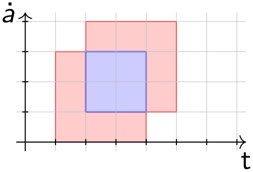
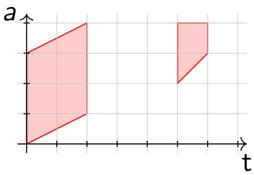


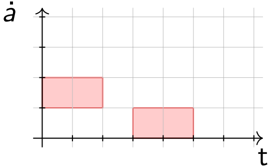


Rules (G_{TDS}), (W_{TVS}), (J_{TDS}) , (S_{TDS}) not shown.

The Calculus of Temporal Influence III

$$\text{(CDL)}\quad \frac{\mathsf{t} \xhookrightarrow{[t_2,t_3],l_1,l_2} a \qquad \mathsf{t} \xhookrightarrow{[t_1,t_2],l_3} \dot{a}}{\text{lderivative}([t_1,t_2],l_1,l_3)}$$





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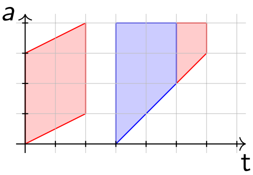

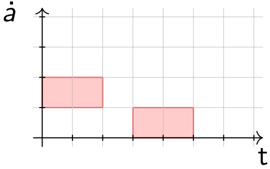

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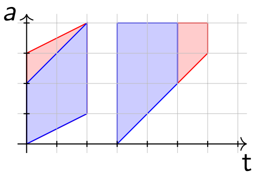

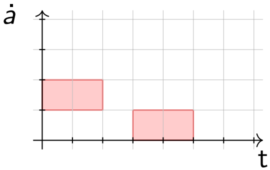

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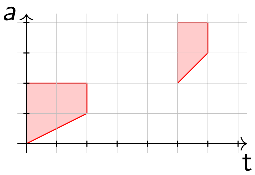

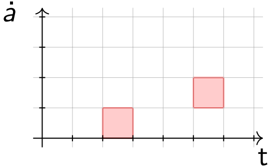

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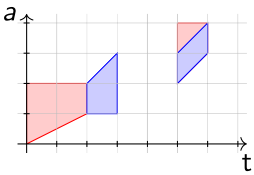

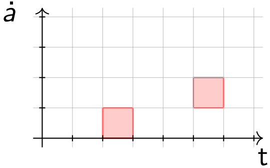

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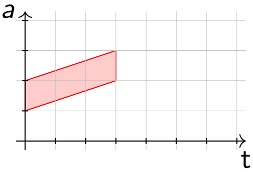

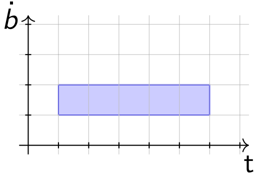

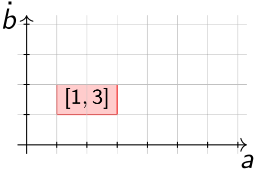

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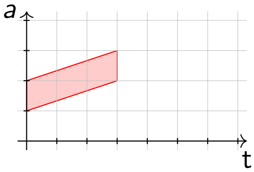
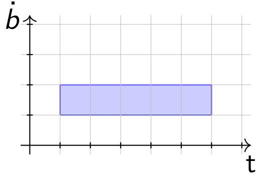
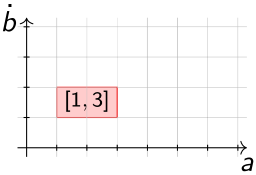
just as VDS, rule (Der) is clearly more complicated than the others







Soundness

Strong Soundness Lemma: all rules preserve logical consequence: if $\mathcal{F} \models T_1$ and $\mathcal{F} \models T_2$ and $\dfrac{T_1 \quad T_2}{S}$ then $\mathcal{F} \models S$

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proof standard; lots of cases

rules are not invertible in general

Theorem 1 (Soundness)

If $\mathcal{C} \vdash S$ then $\mathcal{C} \models S$

PROOF: by induction on height of derivation (standard) □

note: SSL states preservation of countermodels (from conclusion to one premise)

weaker version would suffice for Thm. 1: only preservation of existence of countermodels needed

A Natural Stratification of Proofs

We have soundness, what about completeness?

- need a natural notion of saturation for completeness proof
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normalized scheme intuitively: all statements as tight as possible, no overlaps, information on derivatice included

Lemma 2 (Normalization)

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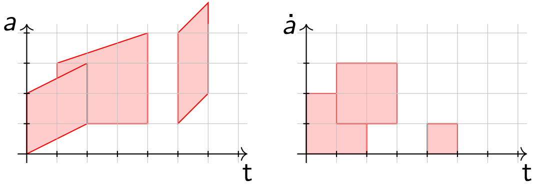

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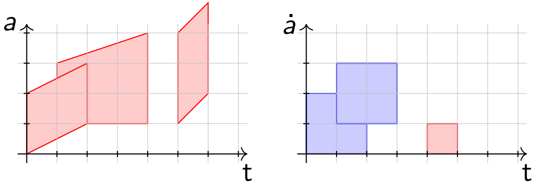

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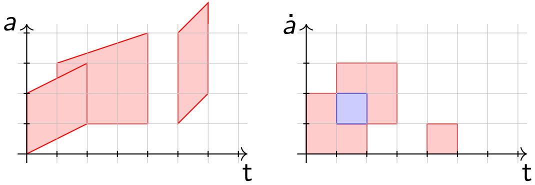

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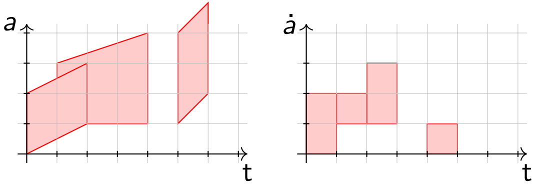

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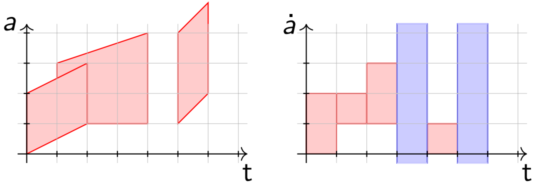

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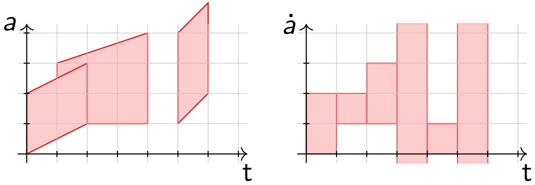

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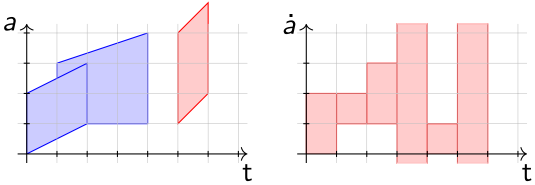

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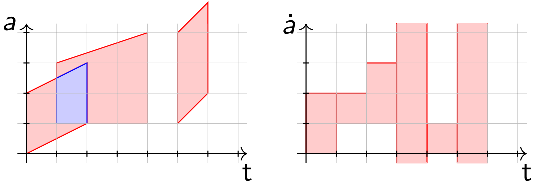

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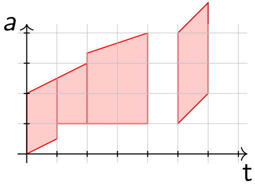

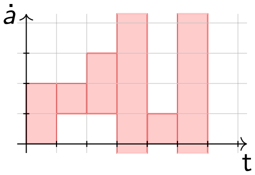

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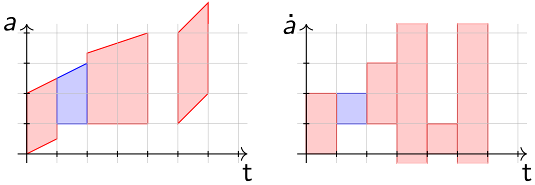

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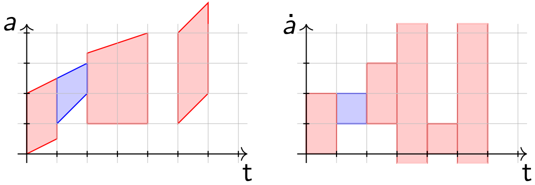

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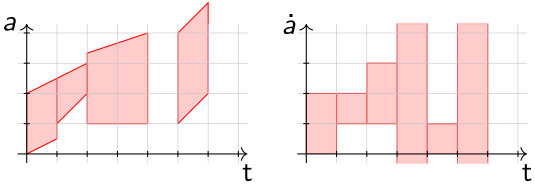

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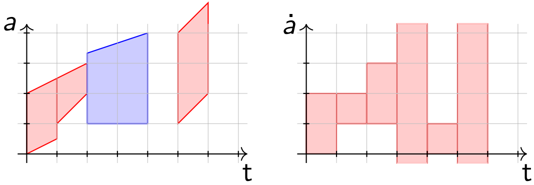

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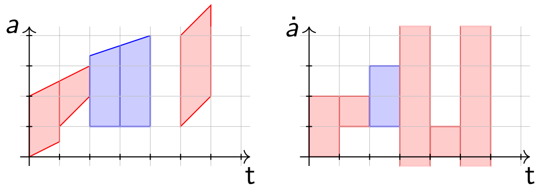

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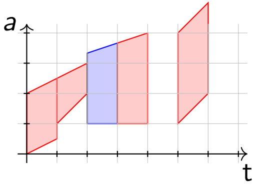

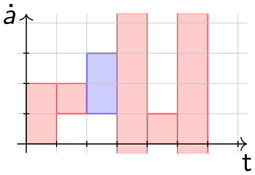

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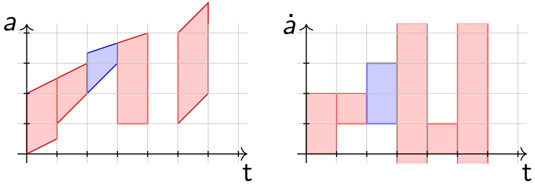

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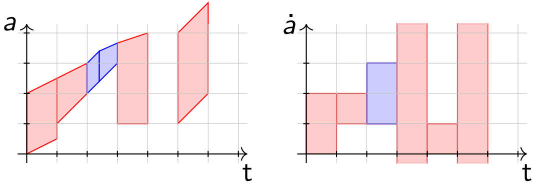

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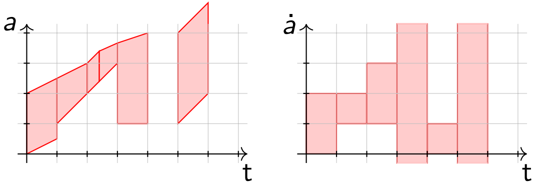

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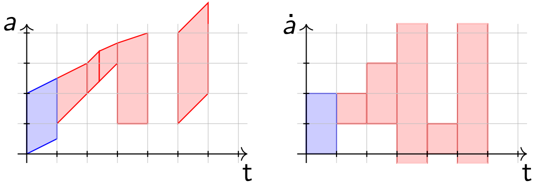

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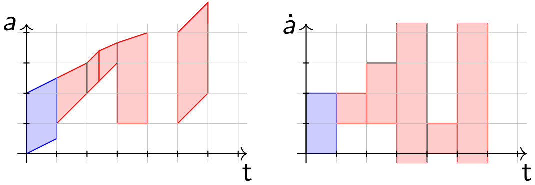

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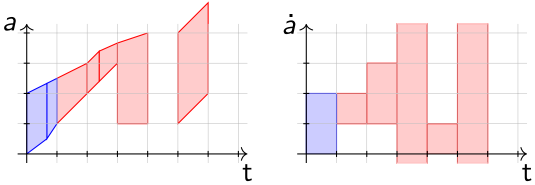

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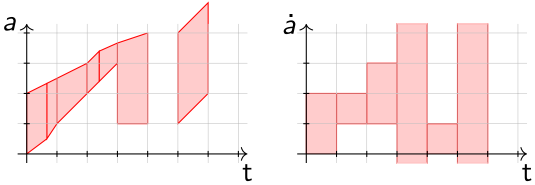

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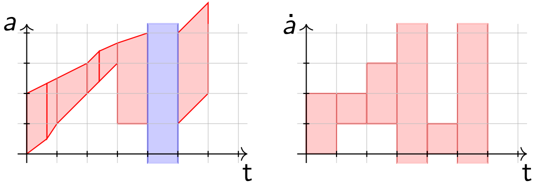

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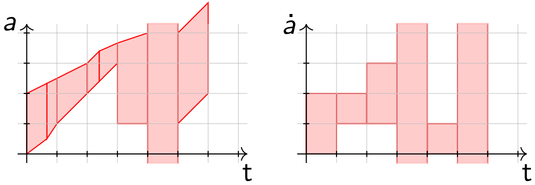

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Proof: Use normalization, then apply rule (Der) to obtain derivable statements for next k

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