Time-aware Robustness of Temporal Graph Neural Networks

Marco Sälzer Silvia Beddar-Wiesing

University of Kassel, Germany

Robustness in the Context of Neural Networks

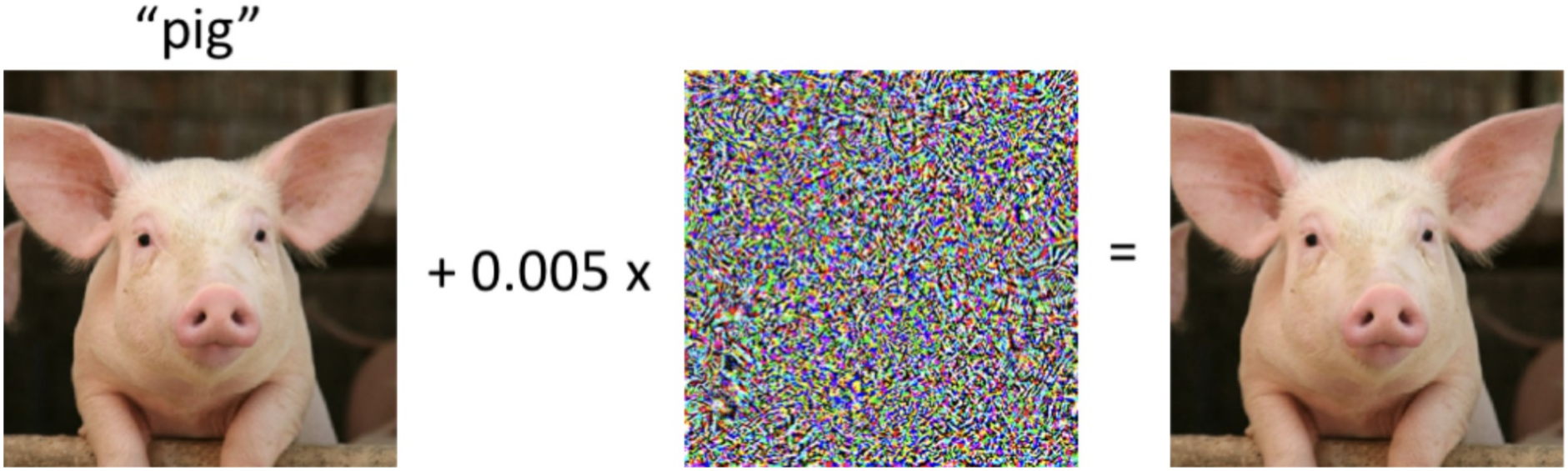
Madry & Schmidt, 2018

Robustness in the Context of Neural Networks

"pig"

Madry & Schmidt, 2018

Robustness in the Context of Neural Networks



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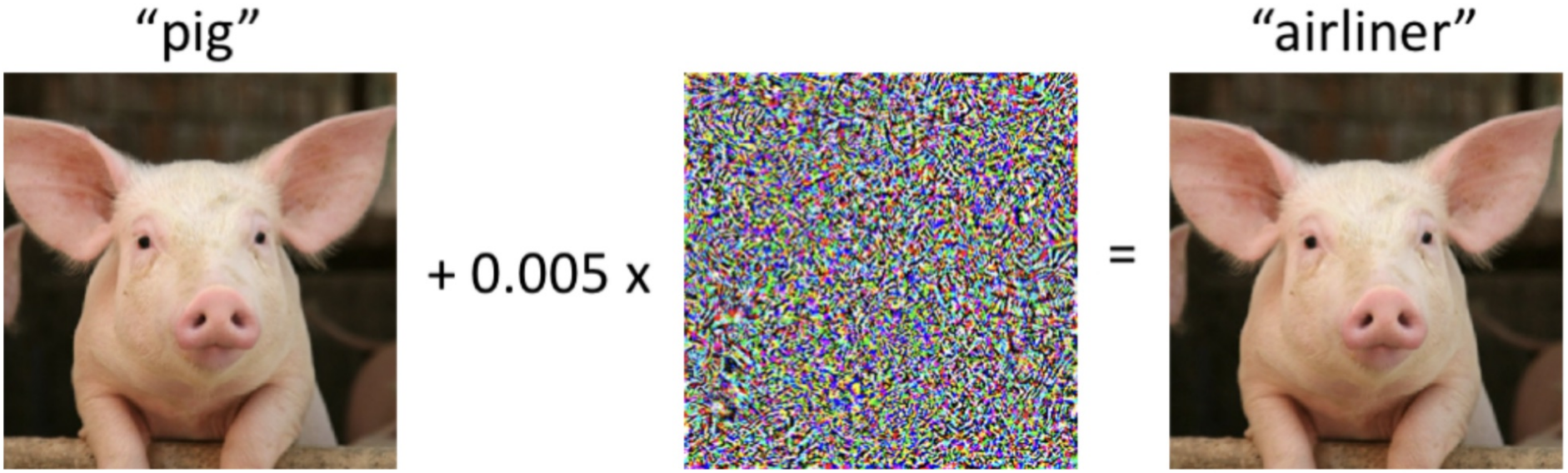

Madry & Schmidt, 2018

Pointwise - Robustness over Real-Valued Data

We call $P = (\bar{x}, B)$ a robustness property where

- $\bar{x}$ is some vector, called center-point
- B is some budget of allowed perturbations

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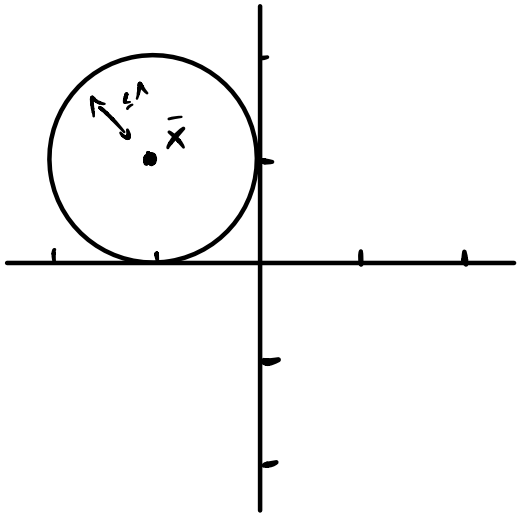

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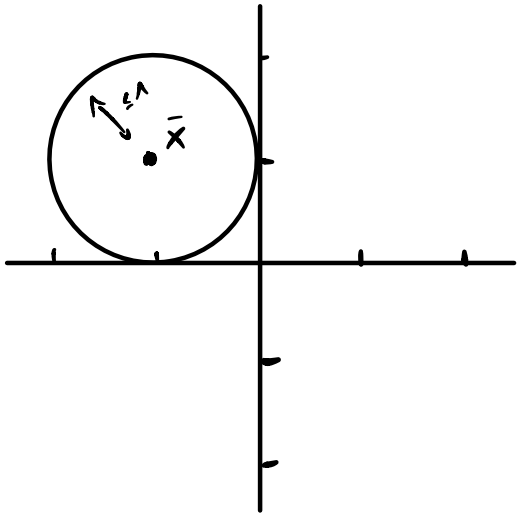
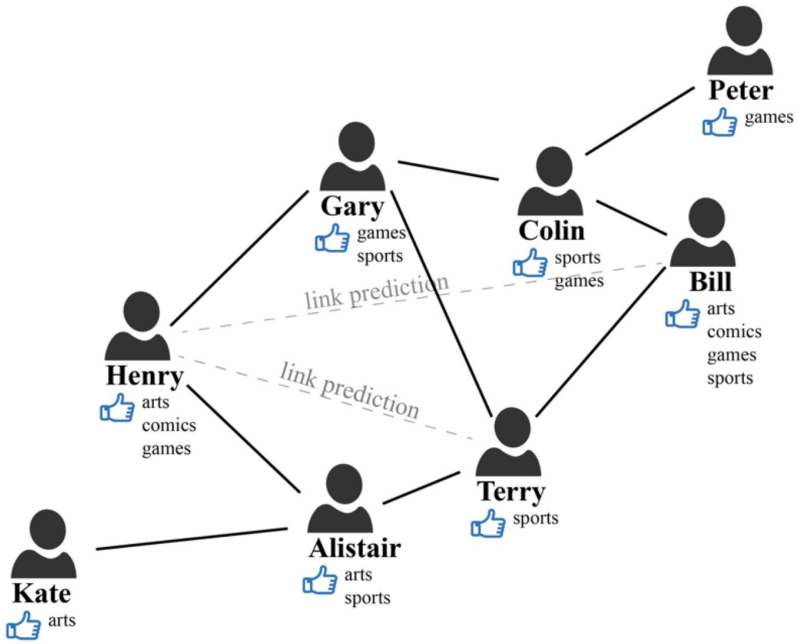
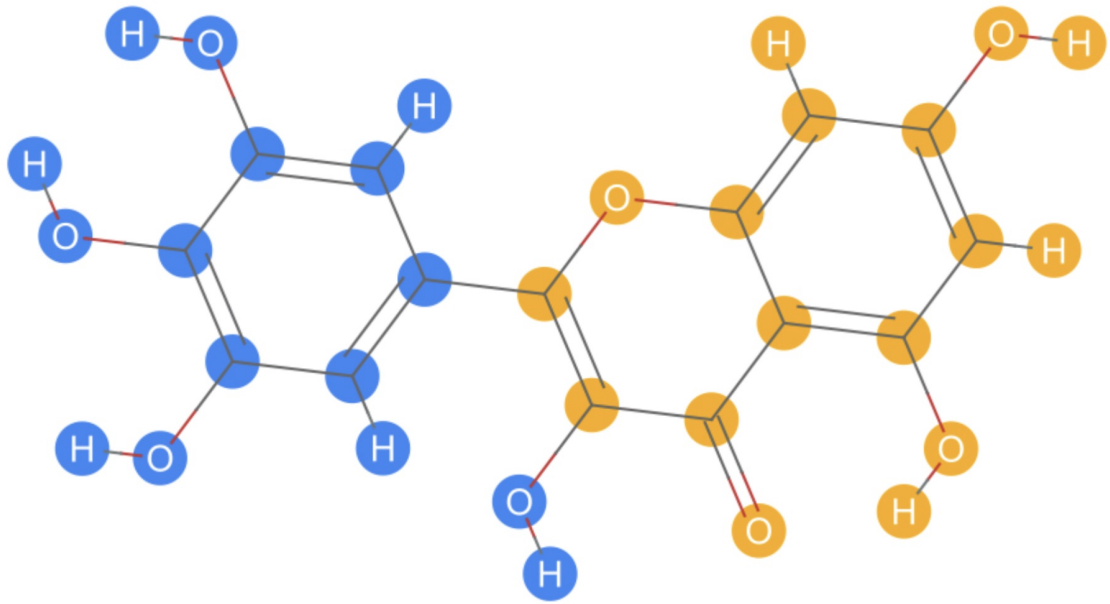


We say that N is robust regarding P if $\forall \bar{x}' \in B$ it holds $N(\bar{x}) = N(\bar{x}')$.

Neural Models for Graph-Data



(1) social networks

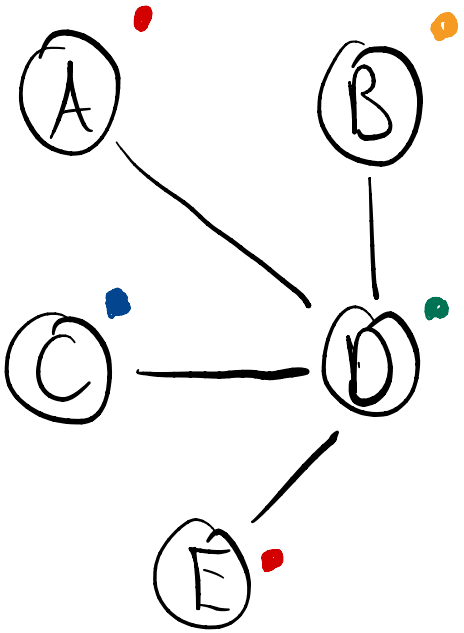


(2) molecule classification

(1) Cuk Baek, 2022
(2) wolfram.com

Graph Neural Networks (GNN)

Usual model: Message-Passing or Aggregation-Combine



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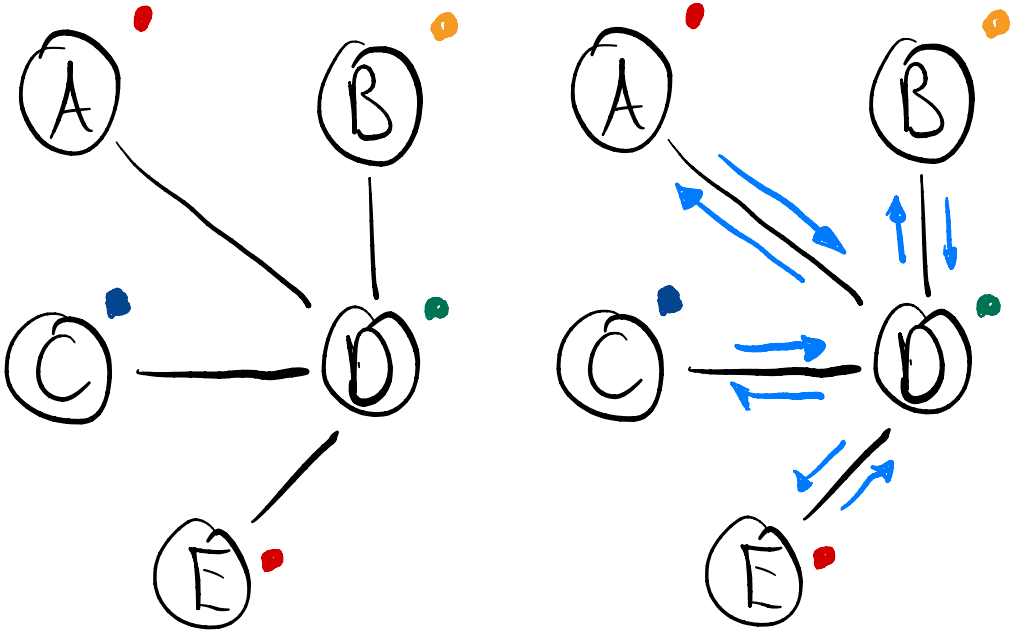

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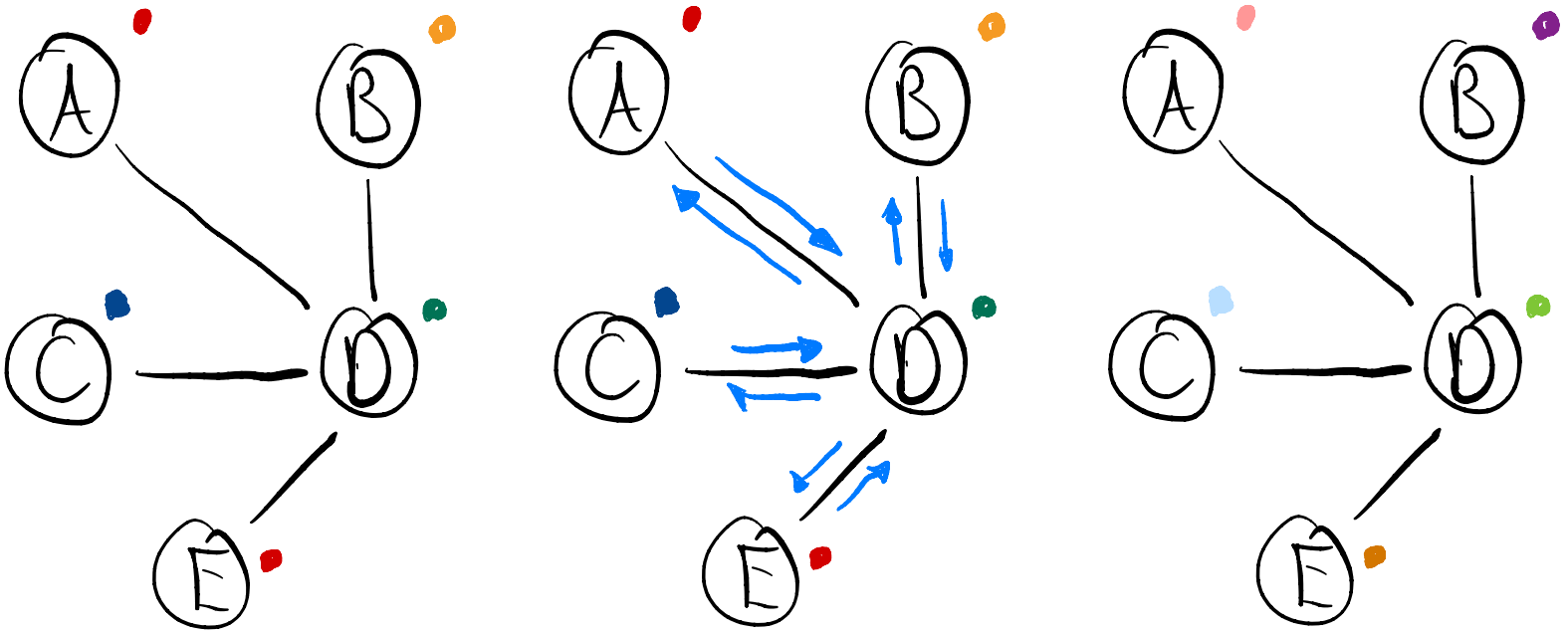

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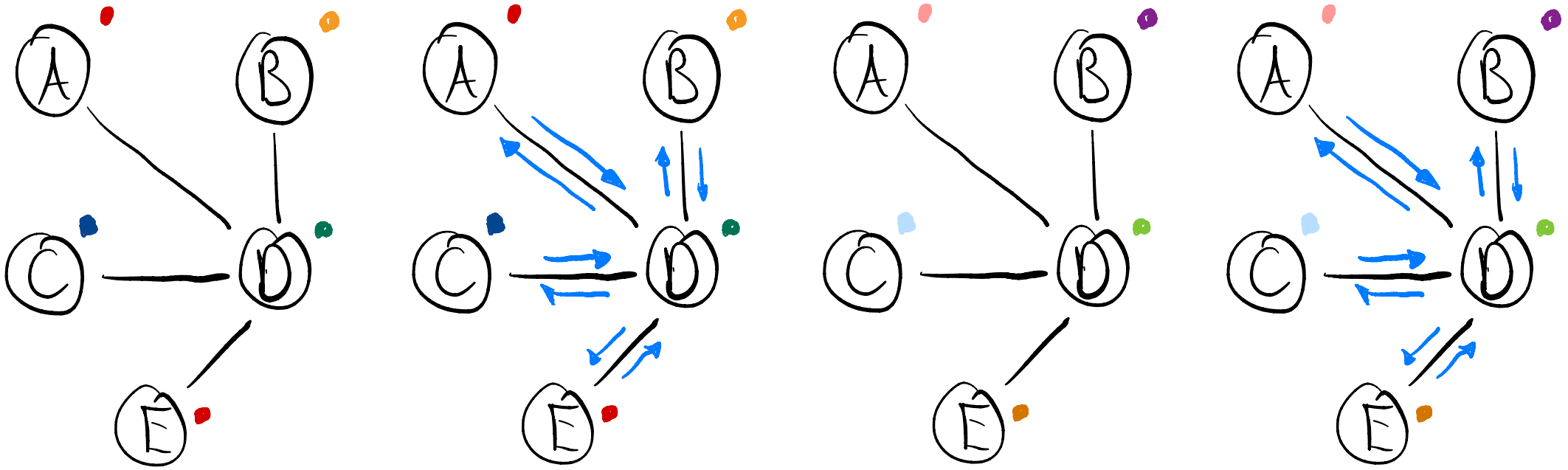

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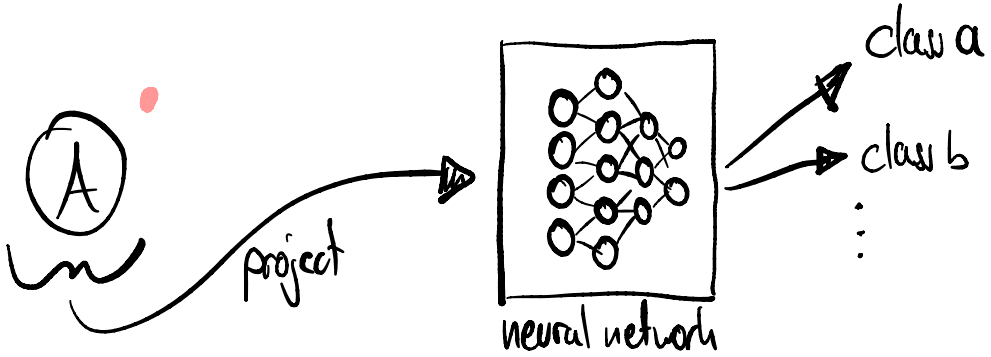

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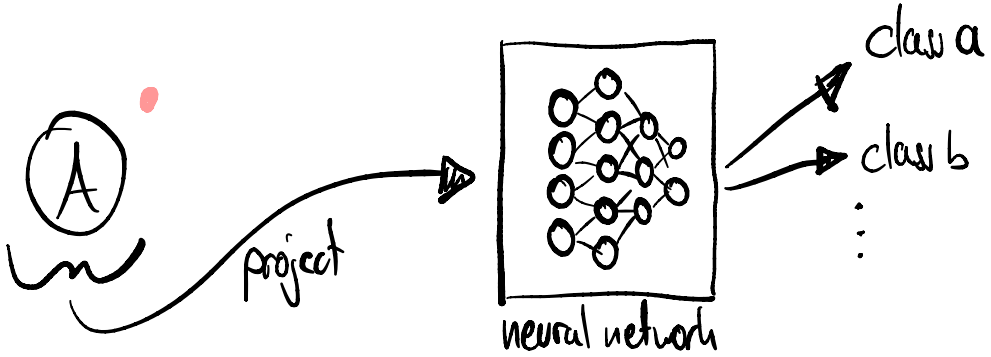
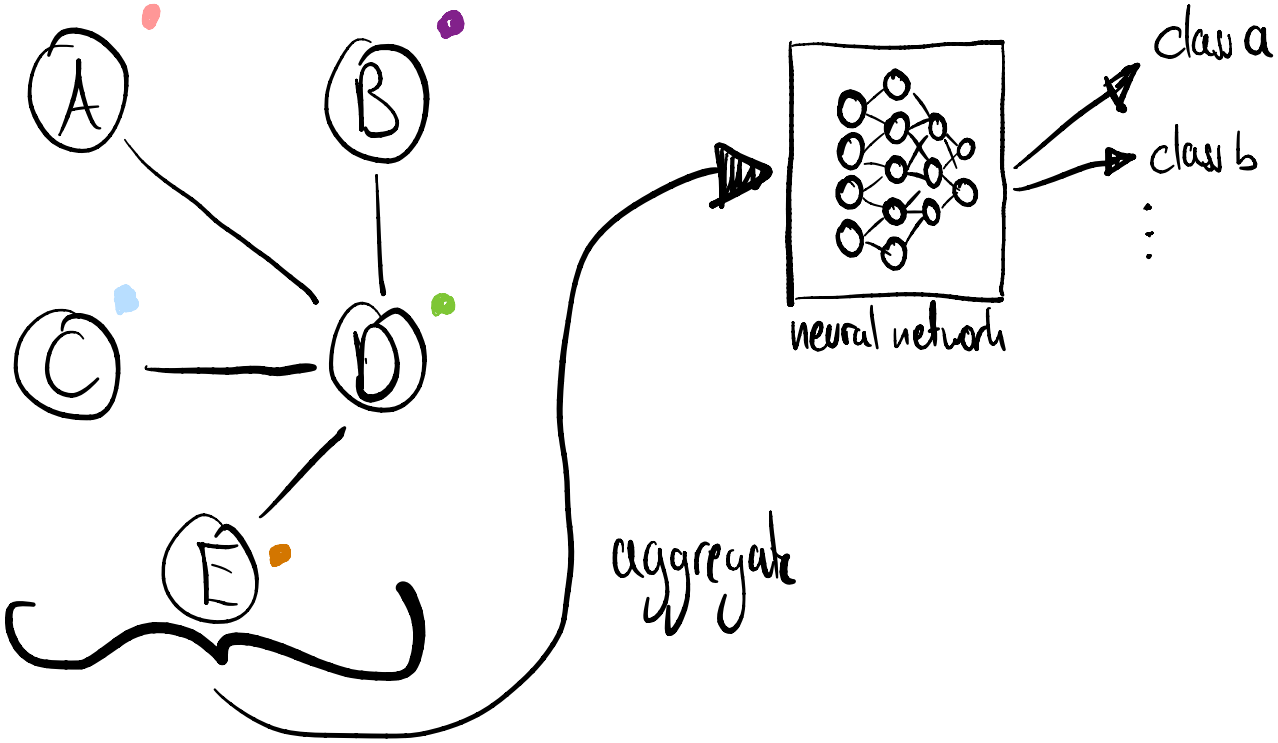


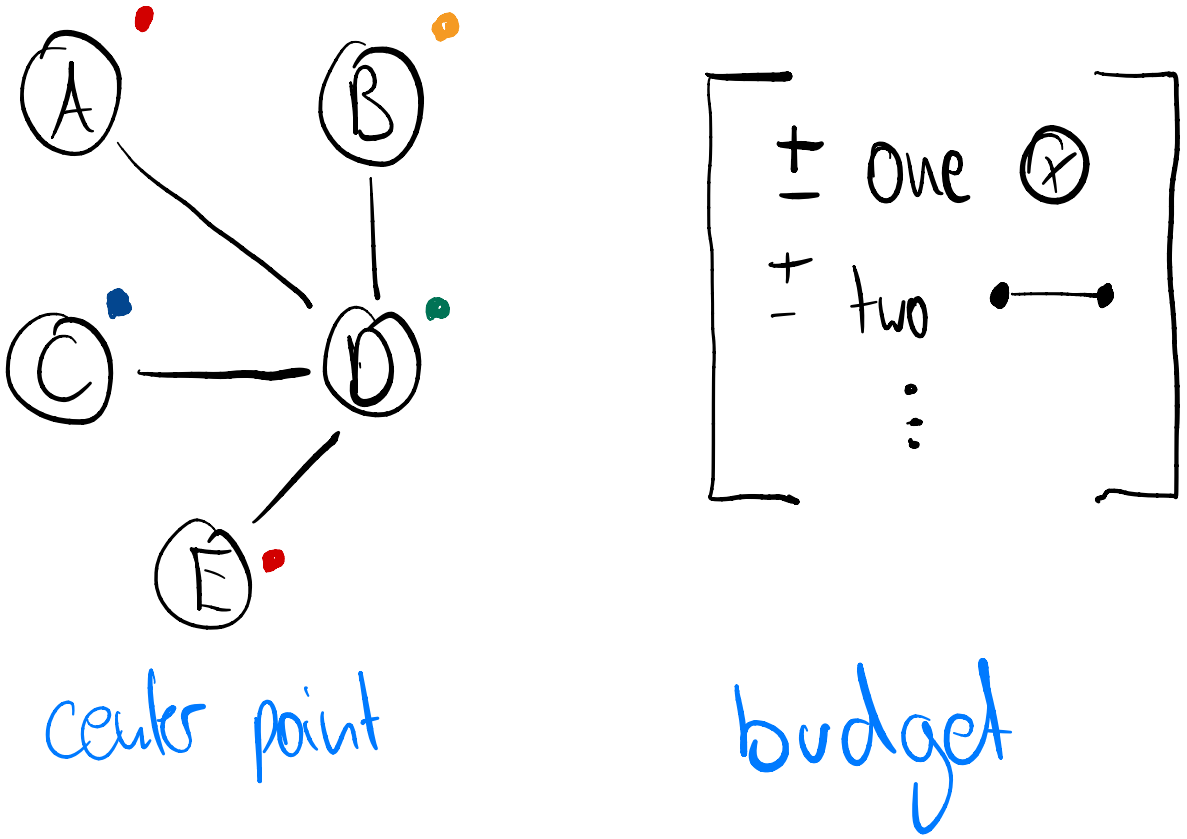
whole graph classification



Point-wise Robustness over Graph Data

Analogous to real-valued data. Center point is some graph G and budget are allowed amount of perturbations.

Example:



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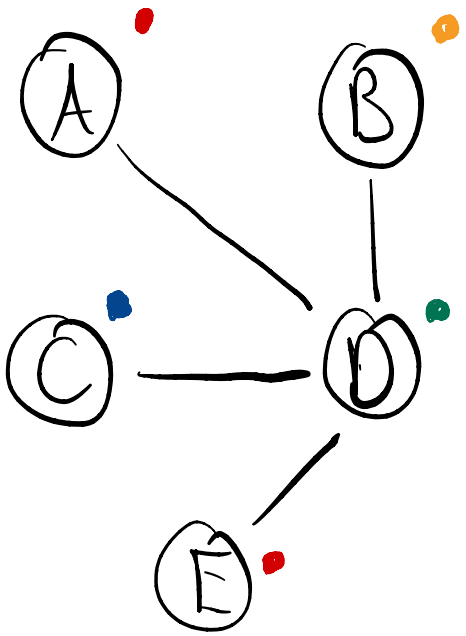

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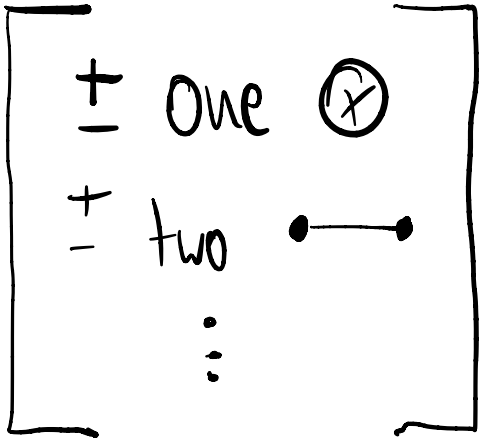
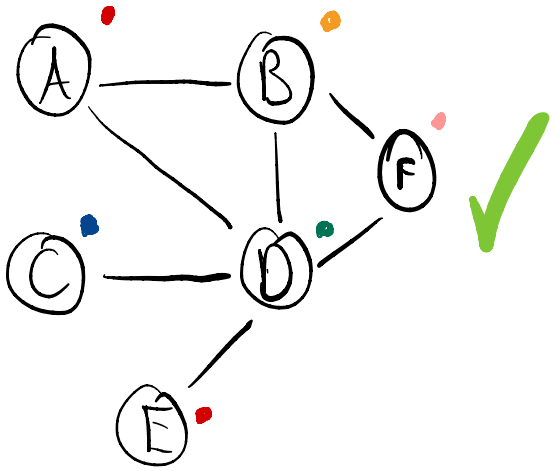


budget



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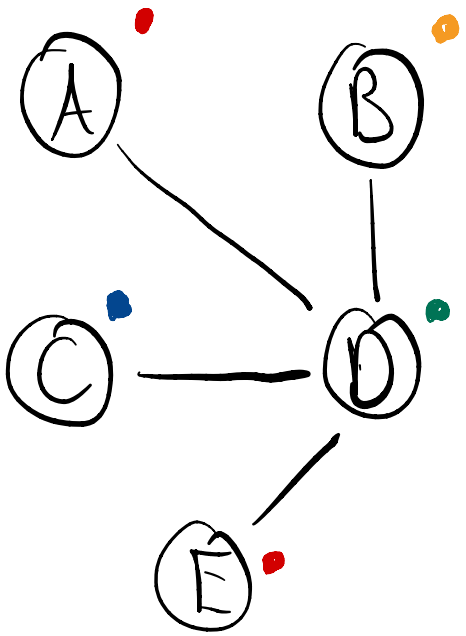

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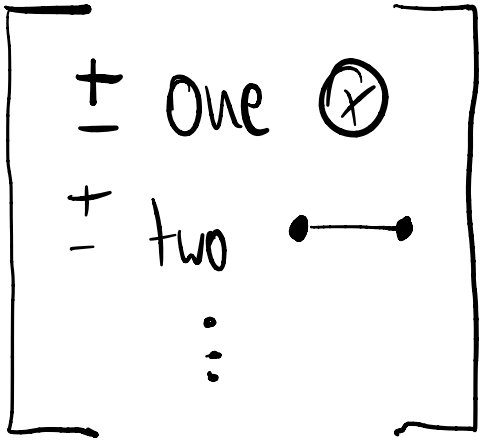

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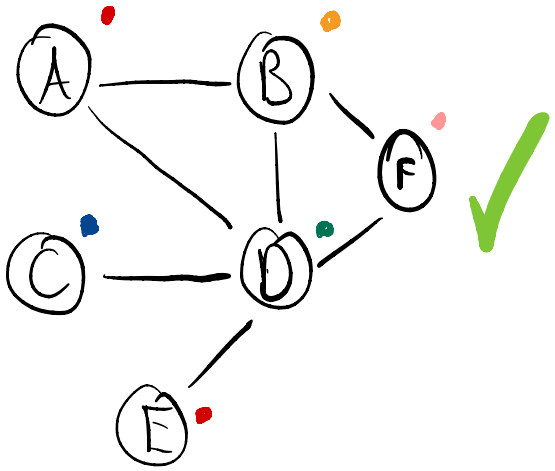
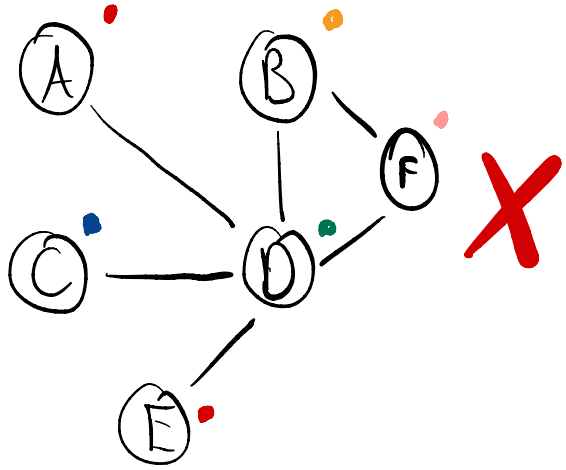
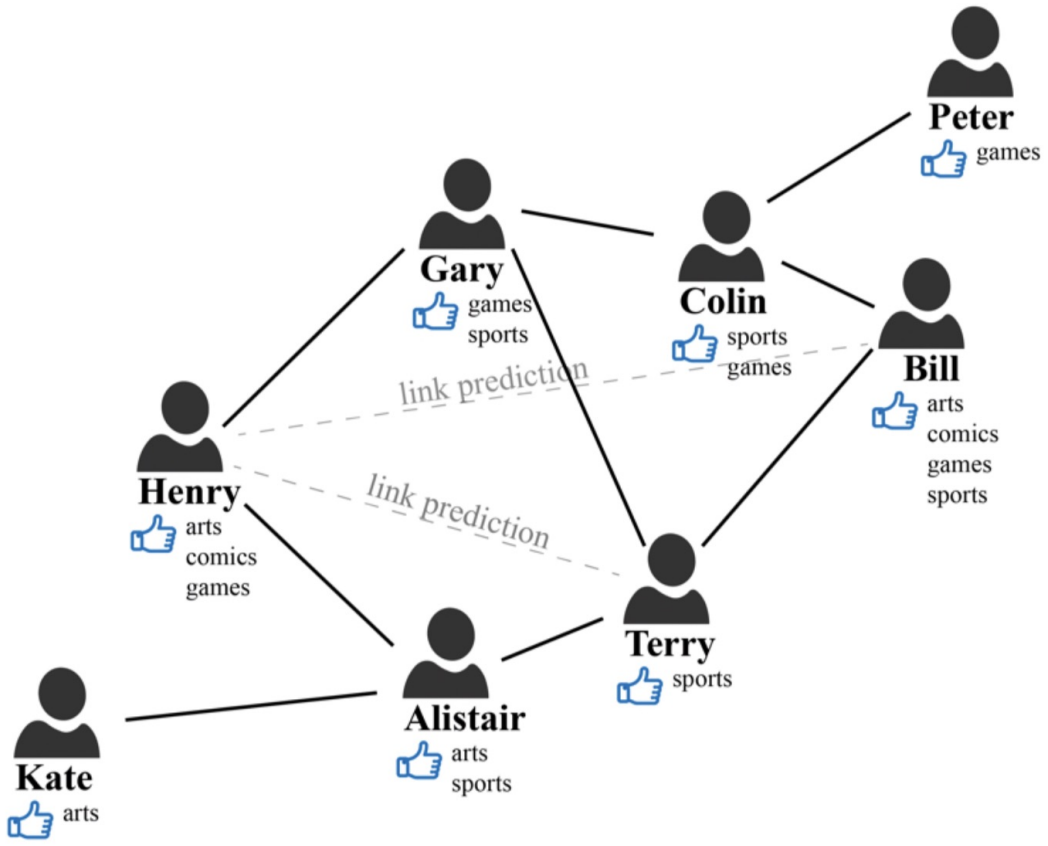




Limitations of the Usual GNN Setting



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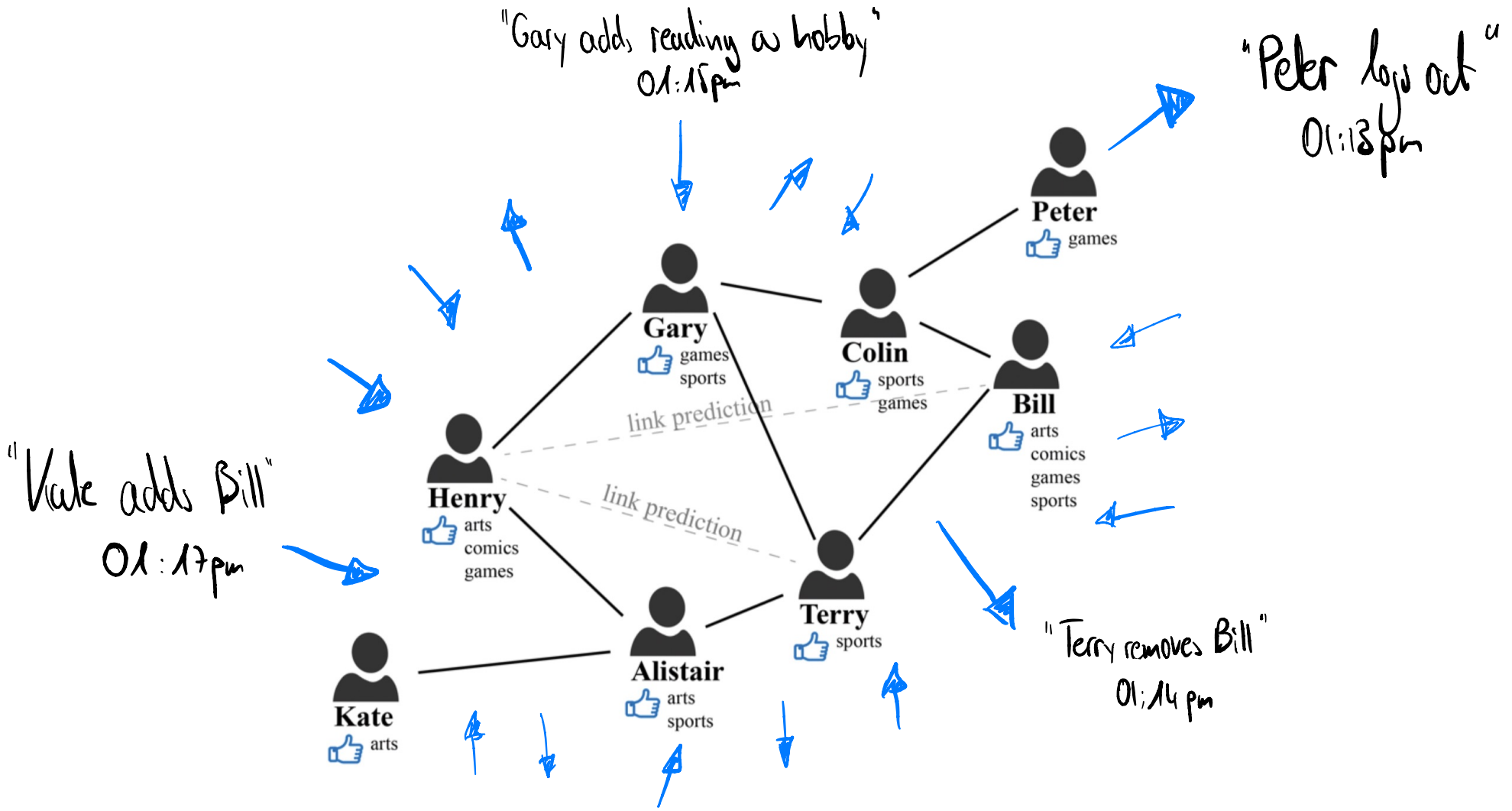

Temporal Graph Neural Networks (TGNN)

A Continuous-Time Temporal Graph (CTG) is a tuple (G, O)

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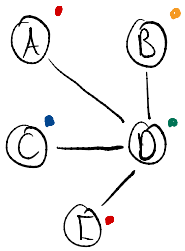
$$\left(\text{  }, \{(t_1, \text{add}, F), (t_2, \text{add}, (D, F)), \ldots\} \right)$$

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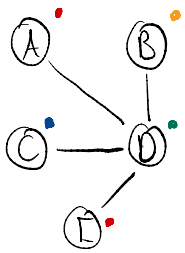
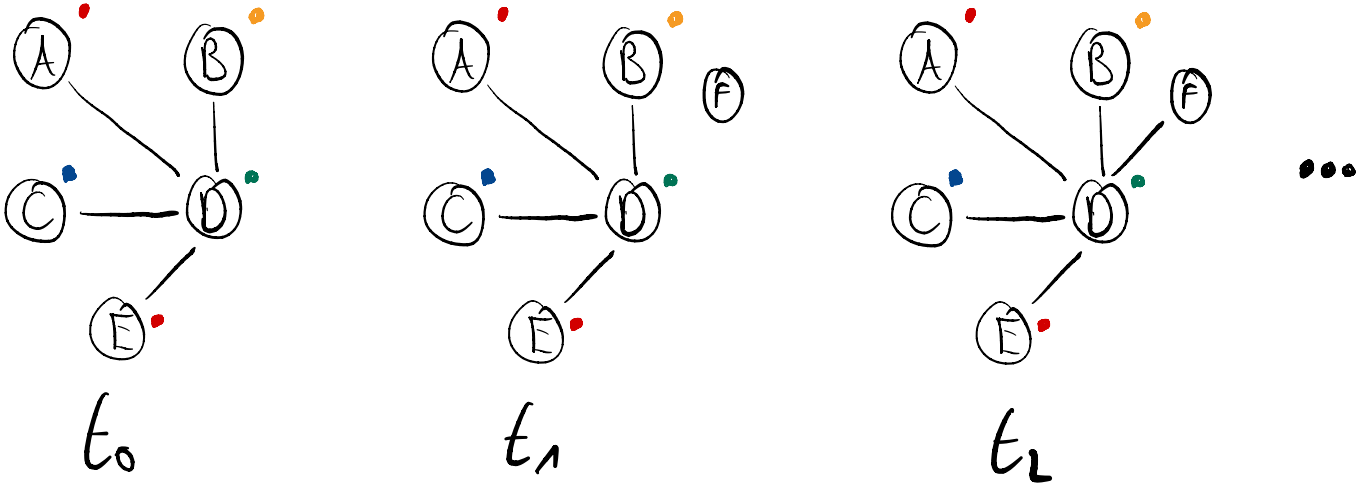
$$\left(\text{  }, \{(t_1, \text{add}, F), (t_2, \text{add}, (D, F)), \ldots\} \right)$$

unfold



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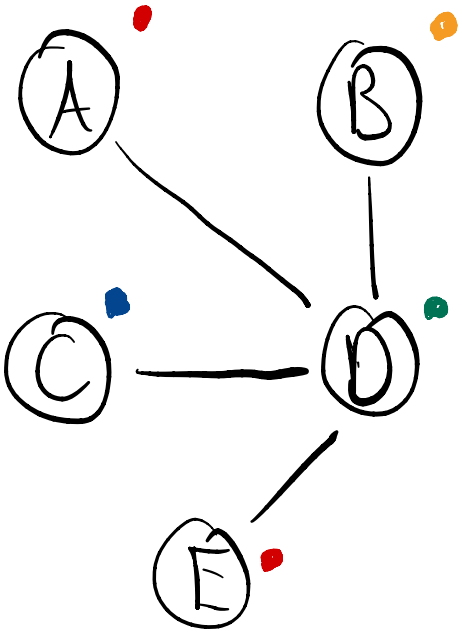

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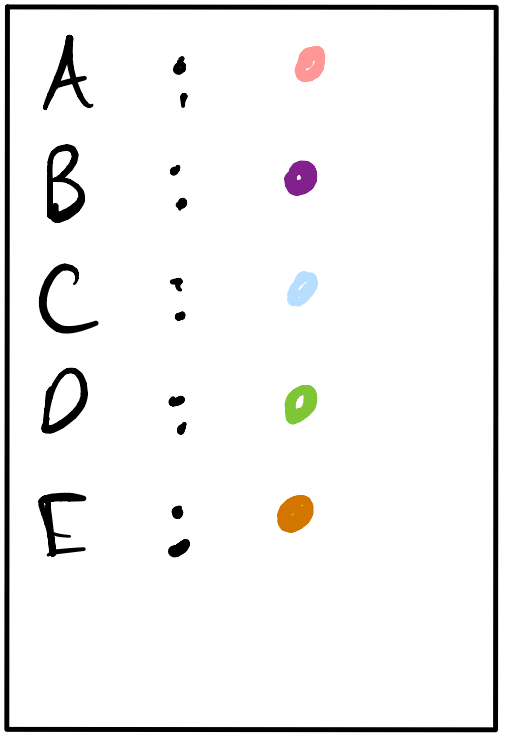

memory

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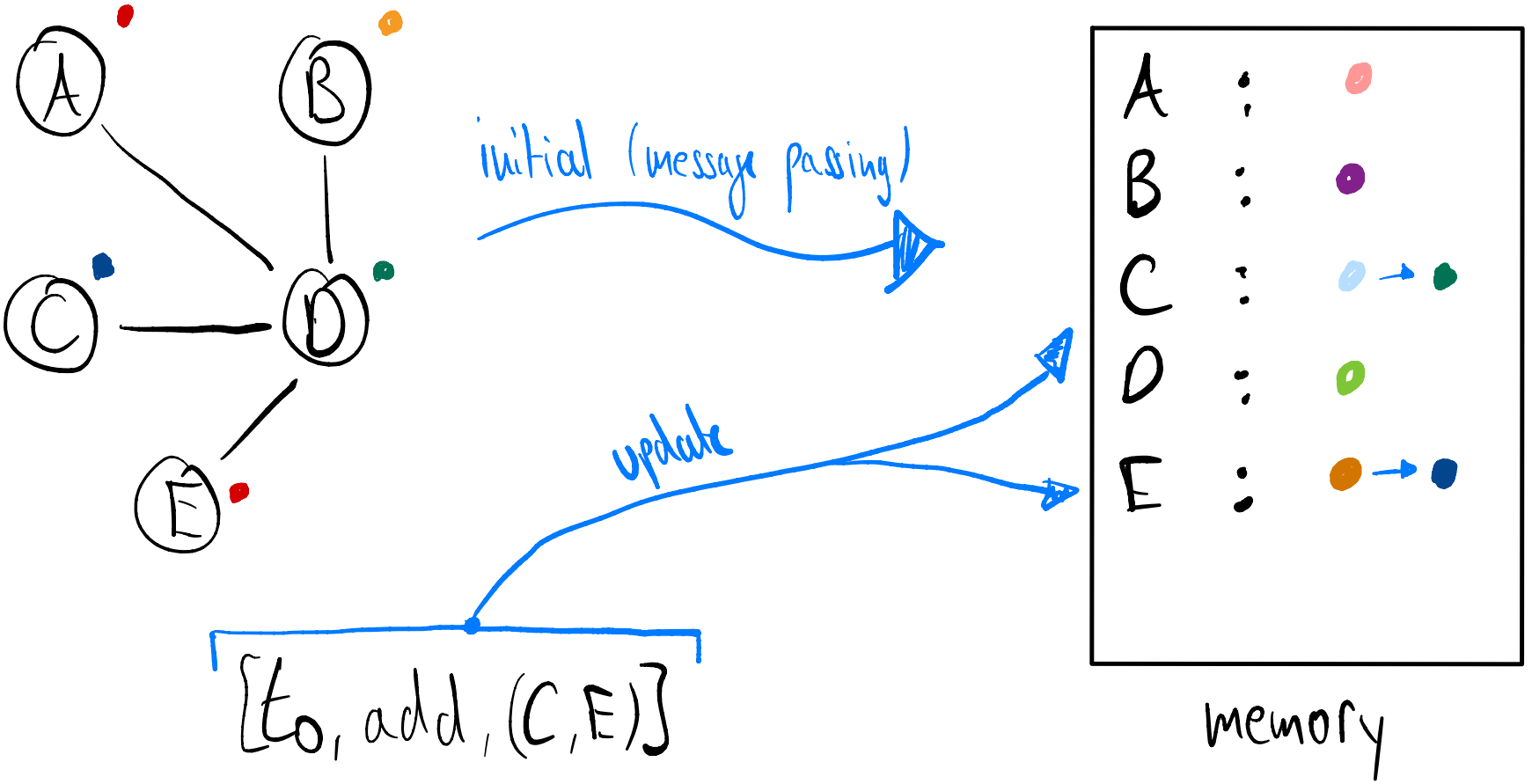

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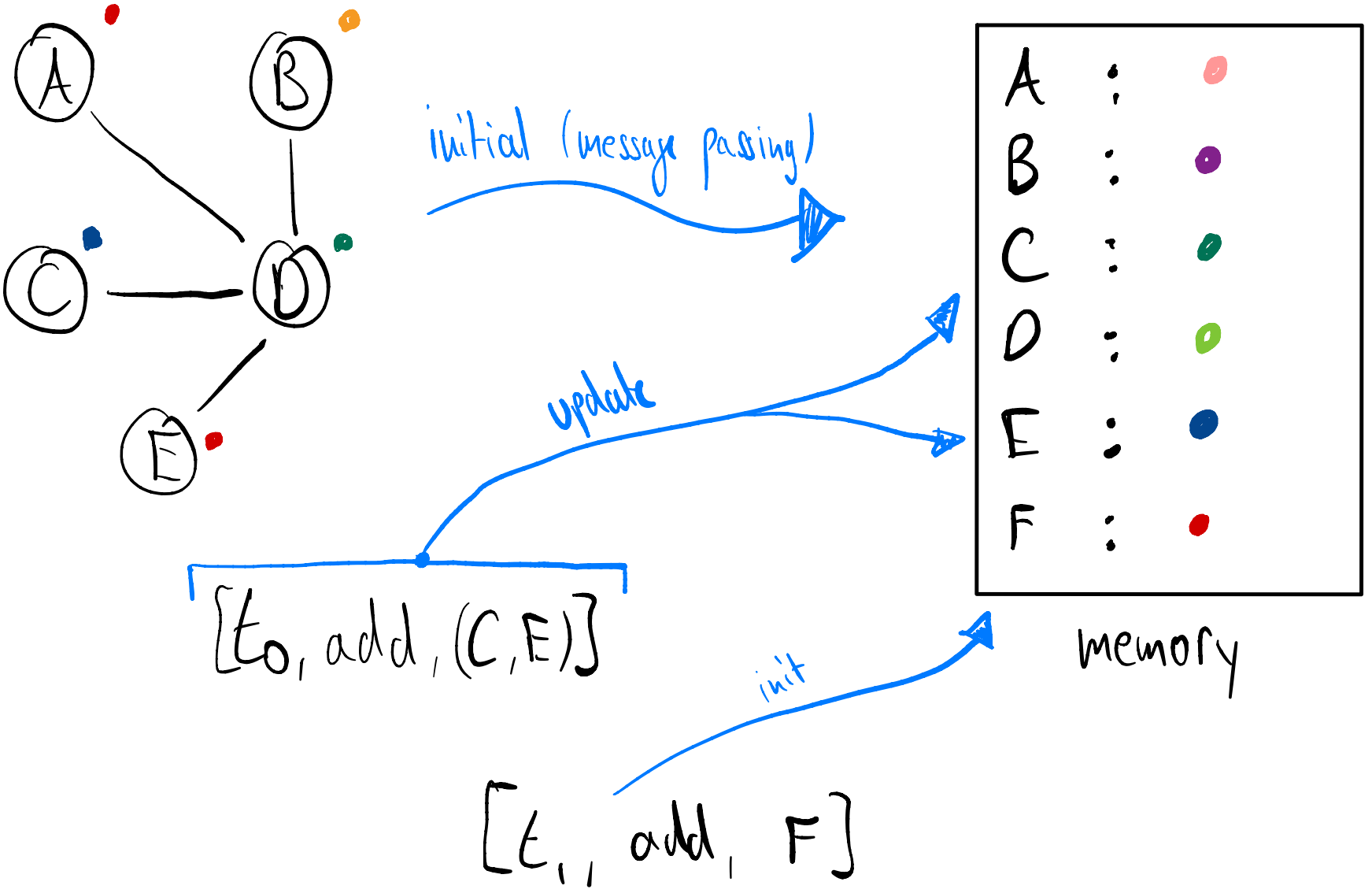

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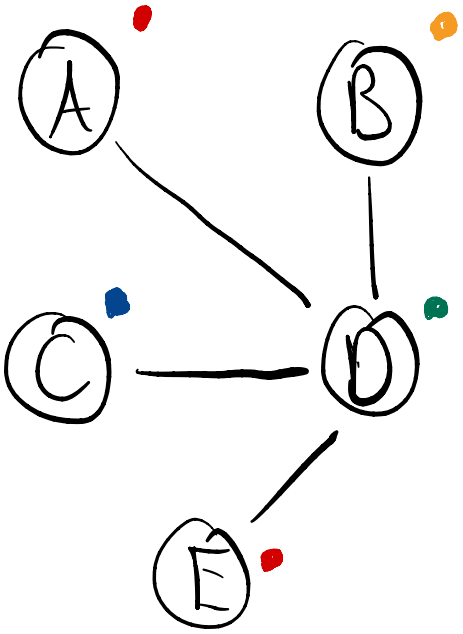

initial (message passing)

update

$[t_0, add, (C, E)]$

init

$[t_1, add, F]$

memory

A :
B :
C :
D :
E :
F :

continuous updates

$[t_2, add, (B, F)]$ ...

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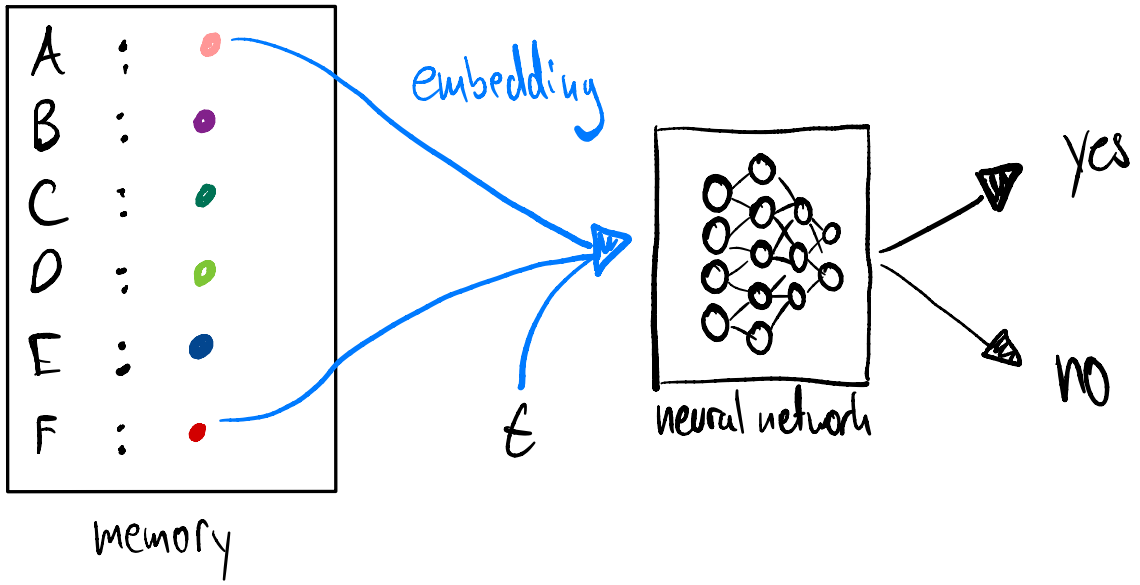
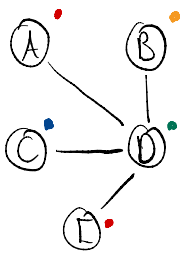


What about Robustness of TGNN?

Point-wise Robustness for Link Prediction over CTG

$$\left(\text{  }, \{(t_1, \text{add}, F), \dots\} \right)$$

center point

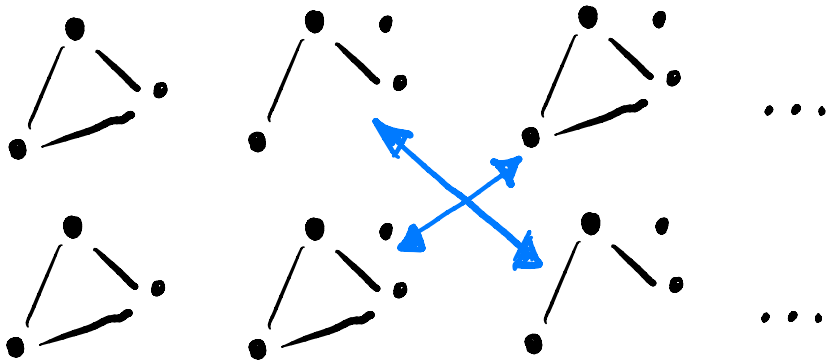
$$\left[\{(t_i', \text{add}, G), \dots\}, \{(t_j, \text{add}, (A,B)), \dots\} \right]$$

budget

"additions" "deletions"

We say that TGNN N is robust for nodes u, v at time t regarding P if for all $C' \in P$ ...

Some Thoughts on Similar CTG



rotations

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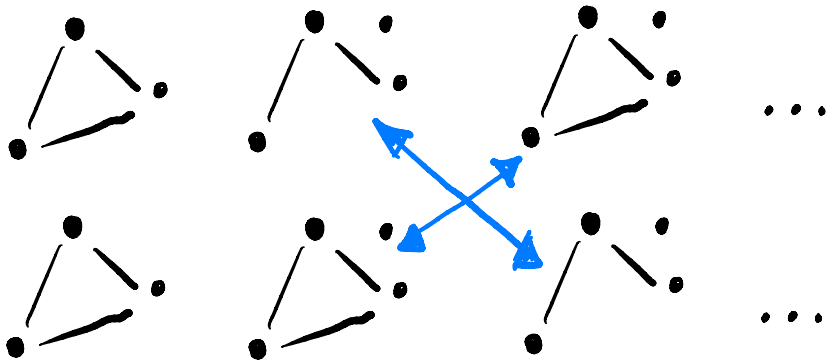

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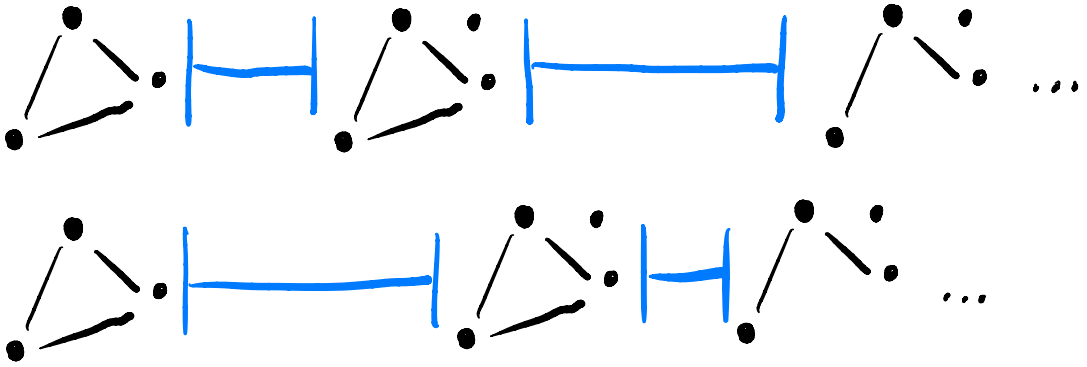

time shifts

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Thanks!