A Decomposition Framework for Inconsistency Handling in Qualitative Spatial and Temporal Reasoning

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Qualitative Spatial & Temporal Reasoning

- A major field of study in KR, and Symbolic AI in general[1]
- Abstracts from numerical quantities of space & time
- Grounded on *physics* and *human cognition*

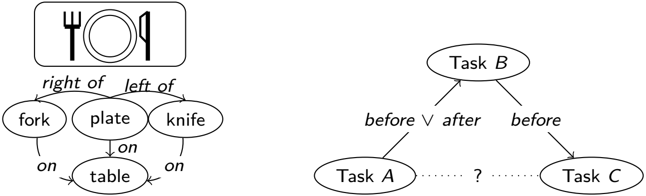

Figure: Abstraction of a spatial configuration (left), temporal constraint network of three variables (right); ? denotes complete uncertainty

[1] G. Ligozat.: *Qualitative Spatial and Temporal Reasoning*. ISTE Series. Wiley, 2011

Example Calculus: RCC8

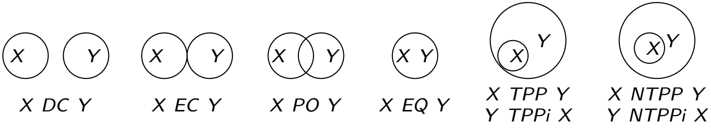

Figure: The base relations of RCC8; $\cdot i$ denotes the inverse of $\cdot$

Example Calculus: Allen's Interval Algebra

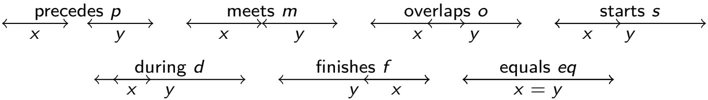

Figure: The base relations of Interval Algebra; inverses are omitted in the figure

Aspects of Space and Time ... and More

- Abundance of calculi dealing with trajectories, occlusion, intervals, and so on[2]

- Translating terminological knowledge into region spaces, e.g., *document PO paper*[3]

[2]F. Dylla et al.: *A Survey of Qualitative Spatial and Temporal Calculi: Algebraic and Computational Properties*. ACM Comput. Surv. 50 (2017)

[3]Z. Bouraoui et al.: *Region-Based Merging of Open-Domain Terminological Knowledge*. In: KR 2022

Applications: Geospatial Semantic Segmentation

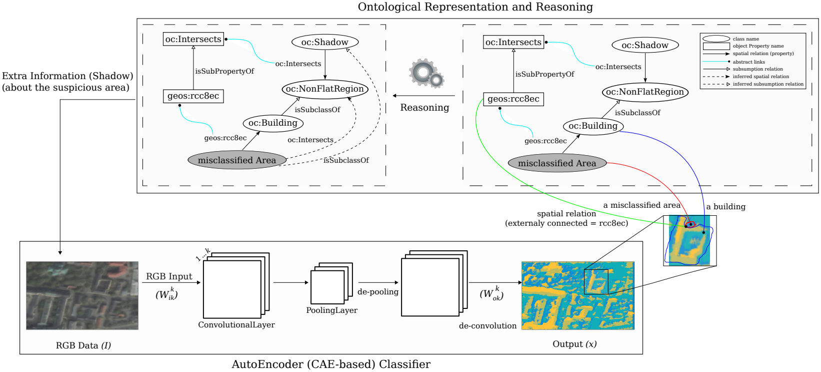

Figure: A semantic referee reasons about the mistakes made by the classifier based on ontological concepts and provides additional information back to the classifier that prevents the classifier from making the same misclassifications[4]

[4] M. Alirezaie et al.: *Semantic referee: A neural-symbolic framework for enhancing geospatial semantic segmentation*. Semantic Web 10 (2019)

Reasons of Inconsistency

- Inaccurate classifiers
- Human error
- Multi-source information
- Vagueness
- Noisy data
- . . .

Framework: Decomposing Inconsistent Information

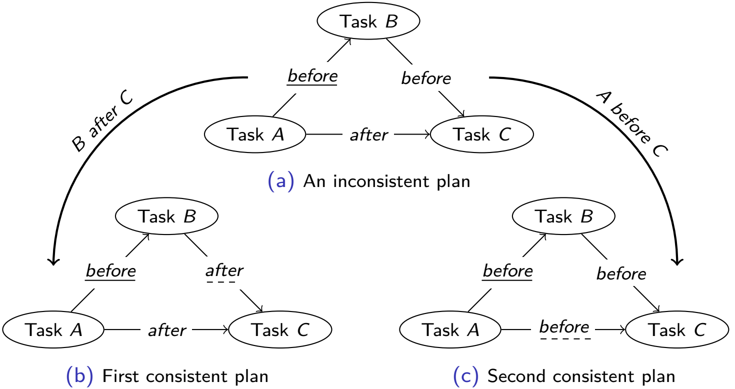

Figure: A decomposition of an inconsistent QCN into consistent components

Complexity

We can obtain the following results

Theorem

For every QSTR formalism $\mathcal{F}$, if SAT($\mathcal{F}$) is NP-hard, then deciding if a QCN *of $\mathcal{F}$ is decomposable into α components is also NP-hard*

Corollary

Deciding if a QCN *of* RCC8 *or Interval Algebra is decomposable into α components is NP-complete*

Complexity: Necessity of Constraints

A set of constraints I may be necessary, i.e., required to be satisfied by all components

Theorem

Deciding if a QCN *of Point Algebra is decomposable into* α *components in the presence of a set of necessary constraints I is NP-complete*

The above theorem may be applied to other polynomial (fragments of) calculi that embed Point Algebra

Optimization Versions

For a QCN $\mathcal{N} = (V, C)$ we consider two optimization problems

- *Minimize number of components*: A maximum of $\lceil |V|/2 \rceil$ components are needed (proof via the Nash-Williams formula)
- *Maximize similarity among components*: A minimum of $\nu(G(\mathcal{N}))$ common constraints can be secured (proof via maximum matching)

We implement greedy constraint methods and optimal Partial MaxSAT encodings to solve these problems

Greedy Constraint Methods: Use of Spanning Trees

For decomposing a QCN $\mathcal{N}$

1. we create a new component of $\mathcal{N}$ by considering a (differentiated) spanning tree of the original QCN

2. we consistently saturate the component with as many of the remaining constraints as possible

3. we rinse and repeat

Optimal Partial MaxSAT Encodings: Exhaustive Search

All necessary constraints in I must be satisfied by all the components

$$\bigwedge_{(i,j)\in I} \bigwedge_{l=1}^{\alpha} \bigvee_{b\in C(i,j)} p_{ij}^{l,b}$$

Each constraint must occur (be satisfied) in at least one component

$$\bigwedge_{(i,j)\in [\![\mathcal{N}]\!]\setminus I} (\bigvee_{l=1}^{\alpha} \bigvee_{b\in C(i,j)} p_{ij}^{l,b})$$

All components must be consistent (atomic + algebraically closed)

$$\bigwedge_{(i,j)\in [\![\mathcal{N}]\!]} \bigwedge_{l=1}^{\alpha} \sum_{b\in \mathrm{B}} p_{ij}^{l,b} = 1$$

$$\bigwedge_{i,j,k\in V, i<j<k} \bigwedge_{l=1}^{\alpha} \bigwedge_{b,b'\in \mathrm{B}} (p_{ij}^{l,b} \wedge p_{jk}^{l,b'} \rightarrow \bigvee_{b''\in b\diamond b'} p_{ik}^{l,b''})$$

Results: Optimal Partial MaxSAT encodings

d	$\text{PMaxSAT}_{\text{min}}$	$\text{PMaxSAT}_{\text{max}}$	$\leq$
4	**2** | 0.55 • **0.01s**	**2** | **0.95** • 0.02*s*	$\frac{0.95}{1}$
6	**2** | 0.45 • **0.02s**	2.0 (3) | **0.97** • 0.12*s*	$\frac{0.97}{1.0\ (2)}$
8	**2** | 0.47 • **0.04s**	2.1 (4) | **0.97** • 37.64*s*	$\frac{0.97}{1.3\ (4)}$
10	**2** | 0.42 • **0.04s**	2.4 (4) | **0.96** • 131.41*s* (15)	$\frac{0.97}{2.5\ (9)}$
12	**2** | 0.37 • **0.03s**	2.6 (5) | **0.96** • 446.67*s* (65)	$\frac{0.96}{4.4\ (10)}$
14	**2** | 0.30 • **0.03s**	? | ? | ? • inf (100)	$\frac{0.93}{8.9\ (15)}$

Table: Assessing the performance of our Partial MaxSAT encodings with Interval Algebra network instances of model A($n = 20, d, l = 6.5$); the format is avg. (max) # of components | avg. similarity • avg. SAT solving time (# of timeouts), plus, in the last column, we present $\frac{\text{theoretical maximum similarity attainable}}{\text{avg. (max) \# of repairs needed}}$ (MAX-QCN)

Results: Greedy Constraint Methods

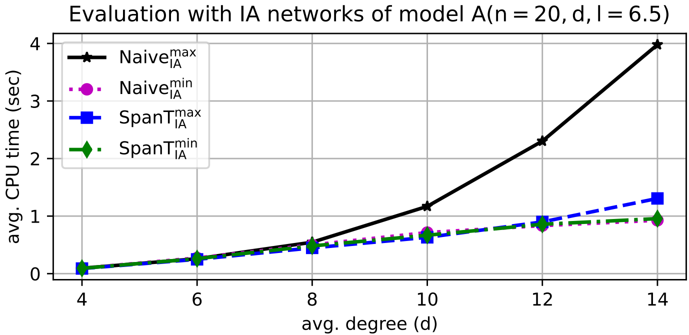

Figure: Assessing the performance of our greedy constraint methods with Interval Algebra network instances of model A($n = 20, d, l = 6.5$) (same as before)

Note: the greedy methods are significantly faster, but lose up to 20% of similarity between components

Inconsistency Measures

For a QCN $\mathcal{N} = (V, C)$ we define two inconsistency measures

- $\mathcal{I}_1(\mathcal{N}) =$ *minimum number of components* for $\mathcal{N}$
- $\mathcal{I}_2(\mathcal{N}) =$ *maximum similarity among components* for $\mathcal{N}$

$\mathcal{I}_1$ and $\mathcal{I}_2$ are similar to $\mathcal{I}_{hs}$[5] and $\mathcal{I}_{mcc}$[6] respectively, satisfying many common postulates

[5] M. Thimm: *On the expressivity of inconsistency measures*. Artif. Intell. 234 (2016)

[6] M. Ammoura et al.: *On an MCS-based inconsistency measure*. Int. J. Approx. Reasoning 80 (2017)

Perspectives and Discussion

- Need of inconsistency-tolerant Hybrid AI systems
- Ranking of different configurations becomes possible via inconsistency measures
- Number of solutions and/or unspecified constraints in a component may be considered
- Tolerating inconsistent components in a decomposition can be explored

Thank you for your interest and attention!

`http://msioutis.gitlab.io`

The purpose of abstraction is not to be vague, but to create a new semantic level in which one can be absolutely precise

Dijkstra