Prime Scenarios in Qualitative Spatial and Temporal Reasoning

Yakoub Salhi[1] **Michael Sioutis**[2]

[1]Université d'Artois, France

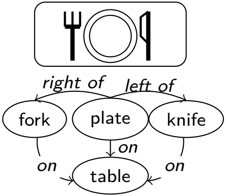
[2]Université de Montpellier, France

26 September 2023

UNIVERSITÉ D'ARTOIS

Qualitative Spatial & Temporal Reasoning

- A major field of study in KR, and Symbolic AI in general[1]
- Abstracts from numerical quantities of space & time
- Grounded on *physics* and *human cognition*



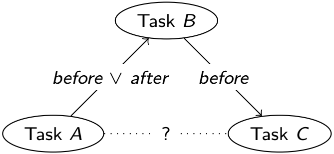


Figure: Abstraction of a spatial configuration (left), temporal constraint network of three variables (right); ? denotes complete uncertainty

.

[1]G. Ligozat.: *Qualitative Spatial and Temporal Reasoning*. ISTE Series. Wiley, 2011

Example Calculus: RCC8

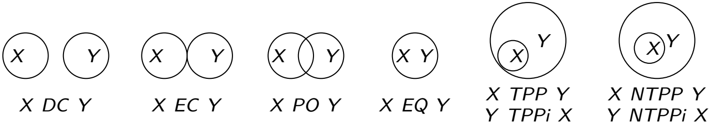


Figure: The base relations of RCC8; $\cdot i$ denotes the inverse of $\cdot$

Example Calculus: Allen's Interval Algebra

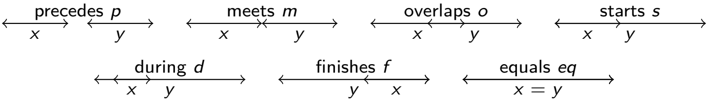


Figure: The base relations of Interval Algebra; inverses are omitted in the figure

Aspects of Space and Time ... and More

- Abundance of calculi dealing with trajectories, occlusion, intervals, and so on[2]

- Translating terminological knowledge into region spaces, e.g., *document PO paper*[3]

[2] F. Dylla et al.: *A Survey of Qualitative Spatial and Temporal Calculi: Algebraic and Computational Properties*. ACM Comput. Surv. 50 (2017)

[3] Z. Bouraoui et al.: *Region-Based Merging of Open-Domain Terminological Knowledge*. In: KR 2022

Applications: Region Approximation

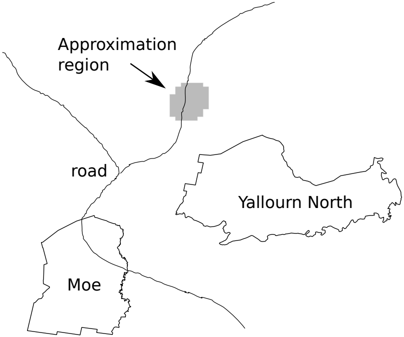


Figure: Illustration of locating a region by natural language descriptions, e.g., "Bushfire burning about 5km northwest of Yallourn North" and "I saw fire about 10km northeast from Moe", with the help of a region approximation method[4]

[4] Z. Long et al.: *Approximating Region Boundaries Based on Qualitative and Quantitative Information*. IEEE Intell. Syst. 37 (2022)

Framework: Prime Scenarios in QSTR

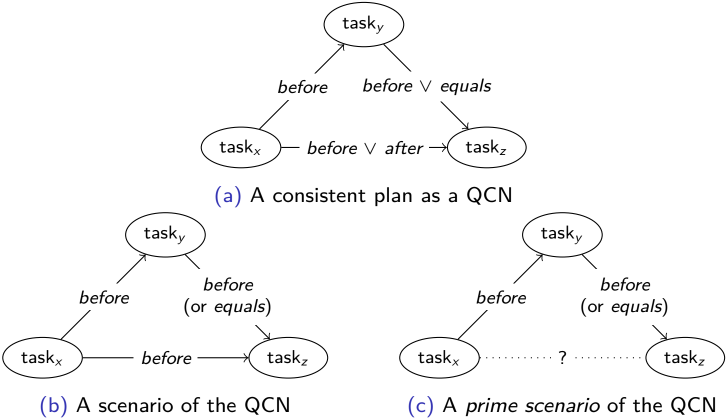


Figure: An illustration of the knowledge compilation notion of *prime scenario* of a qualitative constraint network (QCN)

Framework Extensions

- *Prime scenario cover:* any set of prime scenarios of a QCN that covers all of its (complete) scenarios

- *Minimum-size prime scenarios:* a prime scenario of the smallest possible domain (# of non-universal constraints)

Motivation: XAI and Knowledge Compilation

- Prime scenarios (like prime implicants) can be seen as a sufficient reason behind the decisions of an ML classifier

- Prime scenario covers can be seen as a classical way to perform compilations of spatio-temporal KBs

- Minimum prime scenarios relate to the robustness of a QCN: the smaller the domain of a minimum prime scenario, the more robust the QCN (because of fewer interdependencies)

Computing a Prime Scenario

Algorithm 1: FINDONEPS_1($\mathcal{N}, \mathcal{S}$)

in : A QCN $\mathcal{N} = (V, C)$ and a complete scenario $\mathcal{S}$ of $\mathcal{N}$
output : A prime scenario π that covers $\mathcal{S}$

$\pi \leftarrow \{(i,j) \mapsto b : (i,j) \in [\![\mathcal{N}]\!], b \in \mathcal{S}[i,j], \mathcal{N}[i,j] \neq \mathsf{B}\}$;
for $(i,j) \in [\![\mathcal{N}]\!]$ **do**
  $\mathcal{N}' \leftarrow$ PATHCONSISTENCY($\mathcal{N}^{\pi}_{V\,[i,j]/\mathsf{B}}$);
  if $\mathcal{N}' \subseteq \mathcal{N}$ **then**
    $\pi \leftarrow \pi|_{\mathrm{dom}(\pi)\setminus\{(i,j)\}}$;
return π

FINDONEPS_2 and FINDONEPS_3 optimize the use of the initial input scenario and incorporate dichotomic search respectively

Computing a Prime Scenario Cover

Algorithm 2: $\textsc{ComputePSCover}(\mathcal{N})$

in : A QCN $\mathcal{N} = (V, C)$
output : A PS cover $\mathcal{C}$ of $\mathcal{N}$

$\mathcal{C} \leftarrow \emptyset$;
$\Phi \leftarrow \mathsf{SATEnc}(\mathcal{N})$;
while $\mathrm{SAT}(\Phi)$ **do**
    $\pi \leftarrow \textsc{FindOnePS}(\mathcal{N}, \mathcal{S}_\omega)$;
    $\mathcal{C} \leftarrow \mathcal{C} \cup \{\pi\}$;
    $\Phi \leftarrow \phi \wedge \bigvee_{(i,j)\in \mathrm{dom}(\pi)} \neg p_{ij}^{\pi(i,j)}$
return $\mathcal{C}$

A constraint-based approach can be divised via backtracking and branch exploration in the search tree

Computing a Minimum-Size Prime Scenario (1/2)

Theorem

All prime scenarios can be obtained from the minimal hitting sets of collections of sets built from the counter-scenarios

Finding a minimum-size hitting set as above, gives as a minimum-size prime scenario too

Computing a Minimum-Size Prime Scenario (2/2)

Algorithm 3: $\textsc{MinimumSizePS}(\mathcal{N})$

in : A QCN $\mathcal{N} = (V, C)$

output : A minimum-size prime scenario of $\mathcal{N}$

Let $\mathcal{S}_0$ an arbitrary counter-scenario of $\mathcal{N}$;

$\mathcal{H} \leftarrow \{\mathsf{comp}(\mathcal{N}, \mathcal{S}_0)\}$;

while *true* **do**

  $\pi \leftarrow \textsc{getHS}(\mathsf{MaxSATMH}(\mathcal{H}, \mathcal{N}))$;

  $\mathcal{N}' \leftarrow \textsc{PathConsistency}(\mathcal{N}^{\pi}_{V})$;

  if $\mathcal{N}' \subseteq \mathcal{N}$ **then**

    return π

  Let $\mathcal{S}$ be an arbitrary scenario of $\mathcal{N}'$ where $\mathcal{S}[i, j] \not\subseteq \mathcal{N}[i, j]$ for some $(i, j) \in [\![\mathcal{N}]\!]$;

  $\mathcal{H} \leftarrow \mathcal{H} \cup \{\mathsf{comp}(\mathcal{N}, \mathcal{S})\}$;

Robustness Measure

$$R_{PS}(\mathcal{N}) = max\{|(\llbracket\mathcal{N}\rrbracket| - |\text{dom}(\pi)|) : \pi \in \text{PSes}(\mathcal{N})\}$$

For consistent QCNs, we clearly have

$$R_{PS}(\mathcal{N}) = |\llbracket\mathcal{N}\rrbracket| - min\{|\text{dom}(\pi)| : \pi \in \text{PSes}(\mathcal{N})\}$$

The following properties are satisfied:

Proposition

1. *for any inconsistent QCN $\mathcal{N}$, $R_{PS}(\mathcal{N}) = 0$;*
2. $R_{PS}(\mathcal{N}_\top) = |\llbracket\mathcal{N}_\top\rrbracket|$;
3. *for all two QCNs $\mathcal{N}$ and $\mathcal{N}'$ with $Scenarios(\mathcal{N}) = Scenarios(\mathcal{N}')$, $R_{PS}(\mathcal{N}) = R_{PS}(\mathcal{N}')$;*
4. *for all two QCNs $\mathcal{N}$ and $\mathcal{N}'$ with $Scenarios(\mathcal{N}) \subseteq Scenarios(\mathcal{N}')$, $R_{PS}(\mathcal{N}) \leq R_{PS}(\mathcal{N}')$*

Experimental Findings (1/2)

A preliminary evaluation with QCNs of Interval Algebra of 10 variables was performed

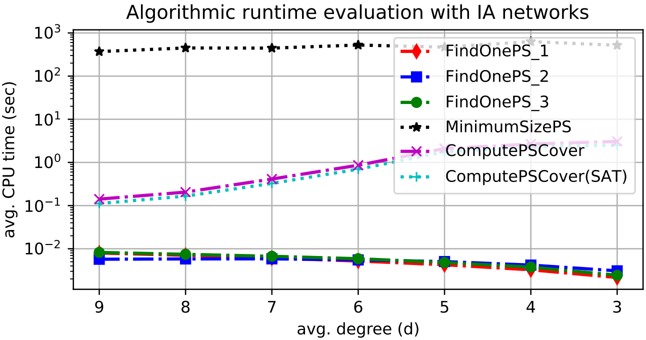


Figure: Assessing the runtime of our algorithms

Experimental Findings (2/2)

- For a prime scenario the *prime index*[5] can be as low as ~ 0.3 for dense QCNs
- Dense QCNs can be covered with as few as ~ 20 scenarios (contrast this to the $O(2^{n \cdot \log n})$ bound)
- Computing a minimum-size prime scenario comes at a huge cost

[5] This is the ratio of the # of non-universal constraints in a prime scenario to the # of non-universal constraints in the original QCN and, thus, takes values in $(0, 1]$

Perspectives and Discussion

- Novel notion of *prime scenario*, analogous to that of prime implicant in classical logic
- Revealing previously unexplored ways to extend the notion of prime implicants to QSTR
- Ranking of different configurations becomes possible via our robustness measure
- A way to explain the decisions made by classifiers compiled into QCNs
- New avenues for research in the field of knowledge compilation in the context of QSTR

Thank you for your interest and attention!

`http://msioutis.gitlab.io`

The purpose of abstraction is not to be vague, but to create a new semantic level in which one can be absolutely precise

Dijkstra